Selected Topics in Physics

a lecture course for 1st year students
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Lecture 9

Introduction to Nuclear Physics

There are Agents in Nature able
to make the Particles of Bodies stick
together by very strong Attractions.
And it is the Business of experimental
Philosophy to find them out.

Isaac Newton, 1730
Opticks, **Book 3**

Plan of Lectures:

1.) Static Properties of Nuclei
1.1) Nuclear charge
1.2) Constituents of nuclei
1.3) Segré chart of stable nuclides
1.4) Nuclear masses
1.5) Nuclear size
1.6) Nuclear binding energy

2.) The Liquid Drop Model of the Nucleus
2.1) The liquid drop model
2.2) The semi-empirical mass formula

3.) Radioactive Decay of Nuclei
3.1) β decay; mass parabolas of isobars
3.2) α decay
3.3) Energy levels
3.4) Spontaneous fission
3.5) Double β decay
3.6) Series of naturally occurring radionuclides

4.) Nuclear Instruments (?)

1.) Static Properties of Nuclei

1.1) Nuclear charge

The chemical elements of matter, ordered by their masses, exhibit periodic chemical and physical properties, as discovered by **D.I. Mendeleev** at the end of the nineteenth century.

The sequential number of an element in Mendeleev's periodic table is called its **atomic number**, denoted by Z.

Thus the atomic number of **hydrogen** is $Z = 1$,

the atomic number of **helium** is $Z = 2$ *etc*.

Elements naturally occurring on earth have atomic numbers up to $Z = 92$ (**uranium**).

Whereas the atomic number was originally merely a sequential number, it was gradually understood, as a result of experiments in the early years of nuclear physics, that it was simply related to the ***nuclear charge.***

Thus, a nucleus of atomic number Z carries a positive charge Ze, where e is the ***elementary charge***:

$$e \cong 1.6 \times 10^{-19} \text{ C}$$

For the atom to be electrically neutral, it must have Z electrons, carrying a charge of $-Ze$.

1.2) Constituents of nuclei

Nuclei consist of tightly bound **protons** and **neutrons**.

A **proton** is the nucleus of the lightest atom, hydrogen. It carries a charge e and has a mass of approximately 1.7×10^{-27} kg.

The **neutron** is electrically neutral; its mass is very slightly greater than the proton mass. In fact, it is considered that this slight mass difference is due to the charge carried by the proton, and otherwise these two particles are identical, in particular they are identical in the purely nuclear forces which they exert on each other. For this reason it is frequently convenient to drop the distinction between them and to call them jointly **nucleon**.

In the language of quantum mechanics one says that the proton and neutron are two different states of the nucleon.

Thus in a nucleus of atomic number Z there are Z protons and N neutrons. The number of neutrons N is called the **neutron number**. The sum of Z and N is denoted by A and called the **mass number**.

Thus, a nucleus of mass number A consists of $A = Z+N$ nucleons.

In a pure sample of some chemical element of atomic number Z one usually finds atoms of slightly different masses. This can be explained as a difference in the neutron numbers, which does not affect the chemical properties.

For this reason one has a special word for an assembly of atoms with equal Z and N: such an assembly is called a **nuclide**.

Nuclides with equal Z but different N are called **isotopes**,

nuclides with equal A but different Z are called **isobars**,

nuclides with equal N but different Z are called **isotones**.

The statement concerning chemical properties is not precise: especially for light elements the differences in the chemical properties can be quite noticeable; they are known as the isotope effect.
But that is a refinement which is outside the scope of the present lectures.

1.3) Segré chart of stable nuclides

On earth, there are **284** naturally occurring nuclides. **16** of these are unstable but have lifetimes exceeding the age of the earth; the latter nuclides are listed in **Table 1**. They will be discussed in detail in Section 3. For the time being we discuss them together with the absolutely stable nuclides.

The ordering of nuclides is displayed in the *Segré chart*, a scatter plot of Z versus N. Every stable nuclide is put into a cell corresponding to its atomic number Z and neutron number N.

The striking feature of this plot is that all nuclides lie in a narrow band close to the line $N = Z$ and with a systematic trend towards values of N exceeding Z with increasing mass number A.
This is the *band of stability*.

Table 1:
Naturally occurring unstable nuclides with lifetimes greater than 10^9 years;
(Source: *BNL Wallet Cards 2000;* http://www.nndc.bnl.gov/)

| Z	Symbol	Name	A	Lifetime years	Decay Modes
19	K	Potassium	40	1.277E9	β⁻ 89.28%, EC 10.72%
23	V	Vanadium	50	1.4E17	β⁻ 17%, EC 83%
37	Rb	Rubidium	87	4.75E10	β⁻
49	In	Indium	115	4.41E14	β⁻
52	Te	Tellurium	123	>6E14	EC
52	Te	Tellurium	130	>5.6E22	2β⁻
57	La	Lanthanum	138	1.05E11	β⁻ 33.6%, EC 66.4%
60	Nd	Neodymium	144	2.25E15	α
62	Sm	Samarium	147	1.3E11	α
71	Lu	Lutetium	176	3.37E10	β⁻
75	Re	Rhenium	187	4.35E10	β⁻
78	Pt	Platinum	190	6.5E11	β⁻
82	Pb	Lead	204	>1.4E17	α (?)
90	Th	Thorium	232	1.4E10	α, SF 1.2E-8%
92	*U*	*Uranium*	*235*	*7.04E8*	*α, SF 7.0E-9%*
92	U	Uranium	238	4.68E9	α, 5.4E-5%
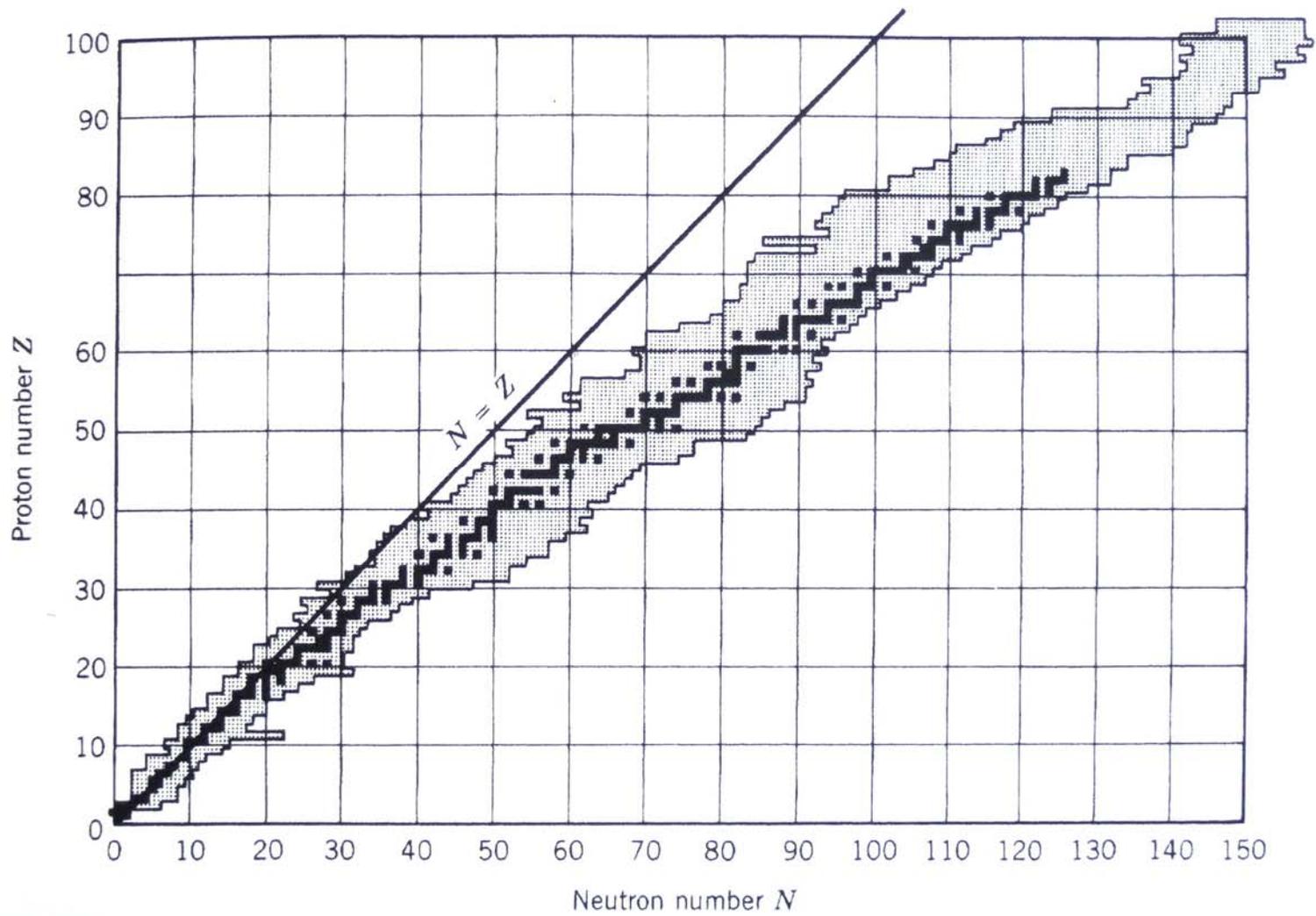
Figure 1.1 Stable nuclei are shown in dark shading and known radioactive nuclei are in light shading.

Segre Chart of Stable Nuclides

N=Z

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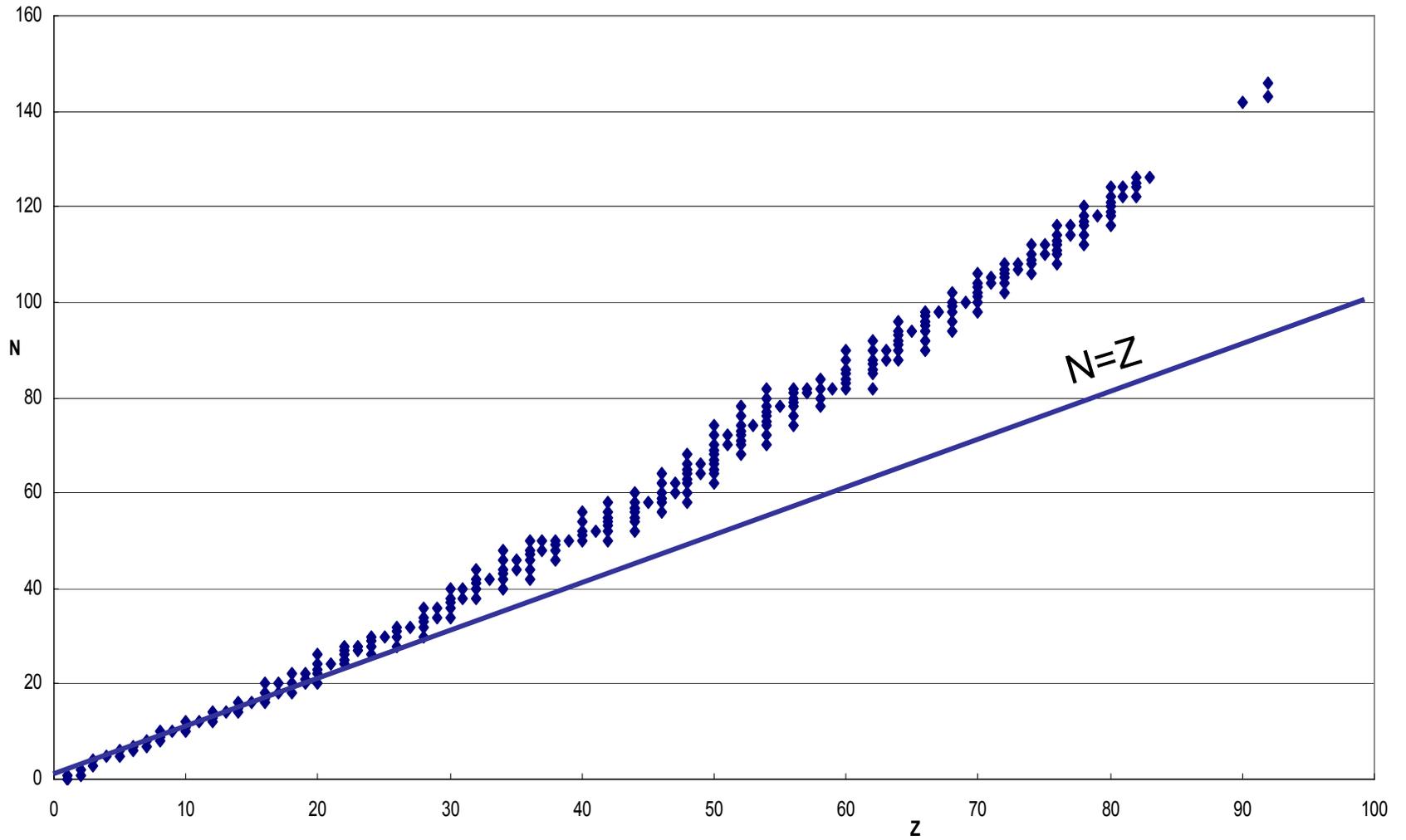
Known unstable nuclides are shown on either side of the band of stability in light shading. These are nuclides which have either an excess or a deficiency of neutrons compared with the stable nuclides. We will discuss this in detail in our section on radioactivity.

In the Segré chart, isotopes are arranged horizontally, isotones vertically, and isobars along lines perpendicular to the line $N=Z$.

We can see that for any given A there are only a few stable isobars.

A detailed analysis yields the following *stability rules:*

166 nuclides have even numbers of Z and N: **even-even** nuclides;

57 nuclides have even numbers of Z and odd numbers of N: **even-odd** nuclides;

53 nuclides have odd numbers of Z and even numbers of N: **odd-even** nuclides;

8 nuclides have odd numbers of Z and N: **odd-odd** nuclides.

Summarizing we can say:
- most abundant are even-even nuclides,
- about equally abundant are even-odd and odd-even nuclides
- and least abundant are odd-odd nuclides.

Four of the odd-odd stable nuclides are light nuclides:

deuterium, lithium, boron and nitrogen;

these are absolutely stable; they have $N = Z.$

The remaining four odd-odd nuclides are unstable but have lifetimes greater then 10^9 years, *i.e.* greater than the age of the earth.

Stability Rules with respect to A:

• Stable nuclides have mass numbers A from 1 to 238;
there is a gap in A from 210 to 231.

• even A nuclides have 1, 2, or 3 isobars, always with even Z.

Three stable even-A isobars exist only at
A = 40, 50, 96, 124, 130, 136, 138, 176, and 180 (see Table 2)

There is no stable even-A nuclide with A = 8

• odd-A nuclides have 1 stable isobar.

Exceptions:
(i)   There is no stable nuclide with A = 5.
(ii) There are two stable nuclides at A = 87, 113, 115, 123, and 187
     (see Table 3).

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Table 2. even-A isobars with 3 stable nuclides

A	Z Symb	Z Symb	Z Symb
40	18 Ar	19 K	20 Ca
50	22 Ti	23 V	24 Cr
96	40 Zr	42 Mo	44 Ru
124	50 Sn	52 Te	54 Xe
130	52 Te	54 Xe	56 Ba
136	54 Xe	56 Ba	58 Ce
138	56 Ba	57 La	58 Ce
176	70 Yb	71 Lu	72 Hf
180	72 Hf	73 Ta	74 W

Table 3. odd-A isobars with 2 stable nuclides

A	Z Symb	Z Symb
87	37 Rb	38 Sr
113	48 Cd	49 In
115	49 In	50 Sn
123	51 Sb	52 Te
187	75 Re	76 Os

(Sb: Antimony; Sn: Tin)

We will gain an understanding of the stability rules when we come to derive the semi-empirical mass formula.

1.4) Nuclear Masses

The unit of mass in nuclear physics is the **_unified atomic mass unit_** u

$$1\,u = \frac{1}{12}\text{ mass of one atom of the nuclide } {}^{12}_{6}\text{C}$$

$$= 1.661 \times 10^{-27} \text{ kg}$$

$$= 931.481 \text{ MeV}\big/c^2$$

In the older literature (before approximately 1960) two other units were used:

$$1\,amu\,(\text{O}) = \frac{1}{16}\text{ mass of one atom of natural oxygen} \quad (chemical\ scale)$$

$$1\,amu\,({}^{16}\text{O}) = \frac{1}{16}\text{ mass of one atom of } {}^{16}\text{O} \quad (physical\ oxygen\ scale)$$

The mass of a nucleus of mass number A is nearly, but not exactly, equal to A unified mass units.

We denote the mass of a nucleus of atomic number Z and mass number A by *M(Z,A)* and then define the ***mass defect*** Δ:

$$\Delta = A - M\left(Z, A\right)$$

Thus, for instance, for iron we have

$$\Delta\left({}^{56}Fe\right) = 56\,\mathrm{u} - 55.9349\,\mathrm{u} = 0.0651\,\mathrm{u}$$

By definition of the unified mass unit, the mass defect of carbon-12 is zero:

$$\Delta\left({}^{12}C\right) = 0$$

For the stable nuclides of mass numbers A < 15 one has Δ<0, and for A > 15 one has Δ>0, and always |Δ|<0.1 u, hence

$$M\left(Z, A\right) \simeq A \gg |\Delta|$$

It is useful to remember that the unified mass unit is nearly equal to, but slightly less than the masses of the proton and neutron.

Here for reference the masses of proton, neutron and of the hydrogen atom expressed in MeV/c^2 and in unified mass units:

$$m_p = 938.272\,\text{MeV}/c^2 = 1.007276\,\text{u},$$

$$m_n = 939.535\,\text{MeV}/c^2 = 1.008665\,\text{u},$$

$$M_H = 1.007825\,\text{u}$$

1.5) Nuclear size

Since the pioneering work of Rutherford it is known that nuclear radii are about ten-thousand times smaller than atomic radii.

All methods of measuring nuclear sizes are based either on *scattering experiments* or on analysis of *nuclear decay*.

Scattering experiments to determine nuclear sizes are of *two kinds*, depending on the type of particle used as a probe, which can be either an *electron* or an *α particle* or a *proton*.

Electrons interact only by *electromagnetic* or by **weak interactions**.

Weak interactions are not relevant to the question of measuring nuclear sizes. Therefore electron scattering experiments are measuring the *charge distributions* of nuclear matter.

Protons, neutrons and α particles are strongly interacting particles.
Therefore these probes measure the nuclear matter distribution.

The two methods yield slightly different results:
the *nuclear matter distribution*, which is determined by the distribution
of protons and neutrons, is different from the *charge distribution*,
determined entirely by the distribution of protons in the nucleus.

Rutherford scattering. KE = 5 MeV, Theta =150 deg
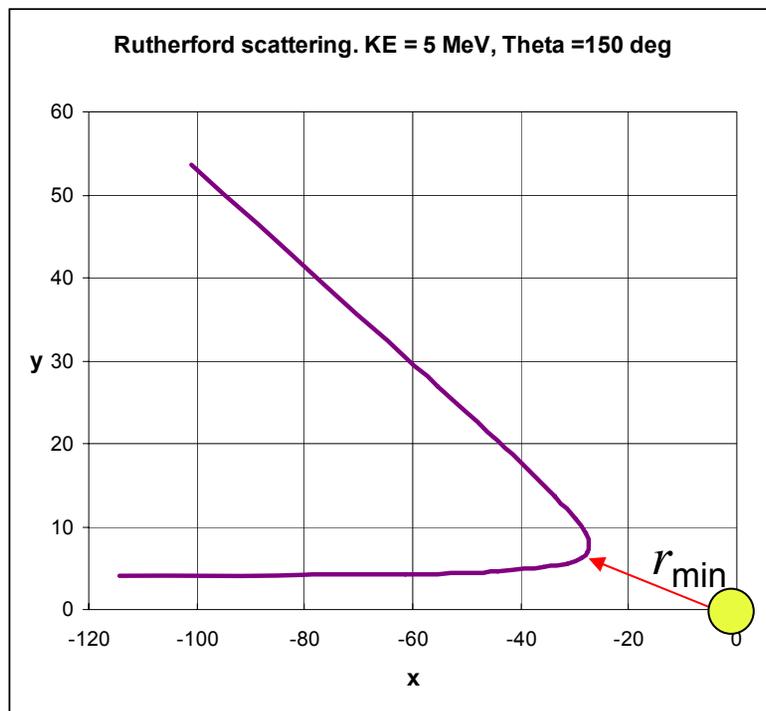
In the Rutherford experiment one measures the distance of closest approach between the α particle and the gold nucleus; this is given by

$$r_{min} = \frac{zZ\alpha\hbar c}{2E}\left(1 + \frac{1}{\sin(\theta/2)}\right)$$

E = KE of the α particle

Moreover, one measures the *angular distribution* of the scattered α particles. This is defined in the following way:
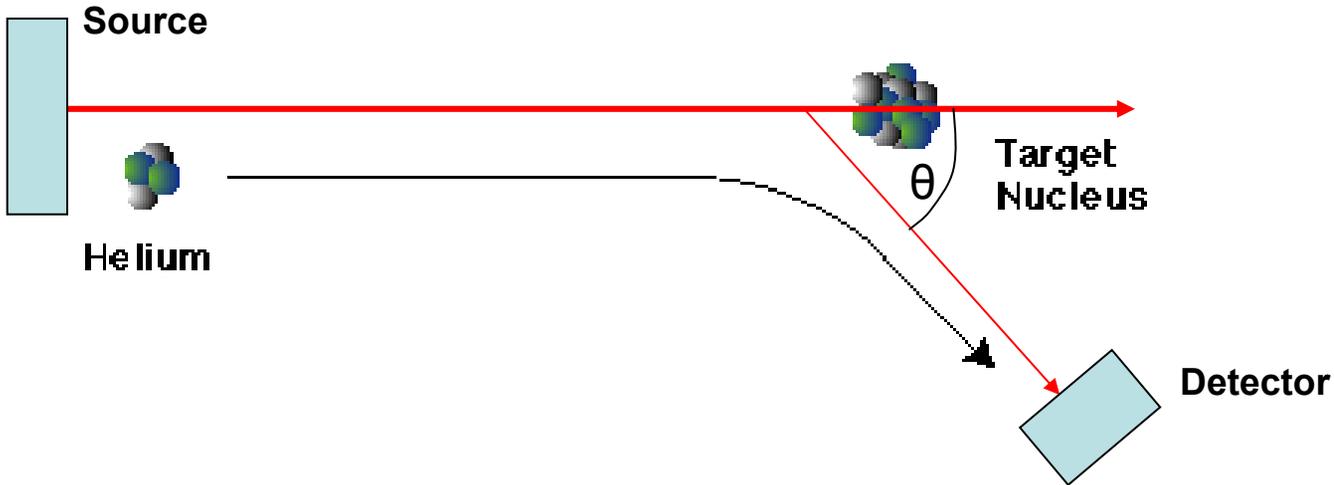
In the incident beam of α particles there are N_0 particles per square cm and second.

Counted in the detector are $N(\theta)d\Omega$ particles per second; the detector is positioned at an angle θ to the beam and subtends a solid angle $d\Omega$ at the centre of force. Then the *differential cross section* $d\sigma$ is defined by

$$d\sigma = \frac{N(\theta)d\Omega}{N_0}$$

The result of the calculation is the following:

$$\frac{d\sigma}{d\Omega} = \left(\frac{\alpha z Z \hbar c}{4E} \right)^2 \frac{1}{\sin^4 \frac{\theta}{2}}$$

Agreement with experiment is excellent as long as the energy is not too large. Depending on the target nucleus, there is an energy at which agreement breaks down. The angular distribution at this and higher energies is called **anomalous Rutherford scattering.**

Anomalous Rutherford scattering sets in when the distance of closest approach is smaller than the radius of the nucleus and hence the α particle comes within the range of the strong nuclear force, *i.e.* the force that holds the nucleus together.

Substituting the energy just below the onset of anomalous Rutherford scattering into the formula for the distance of closest approach one gets therefore the size of the nucleus.
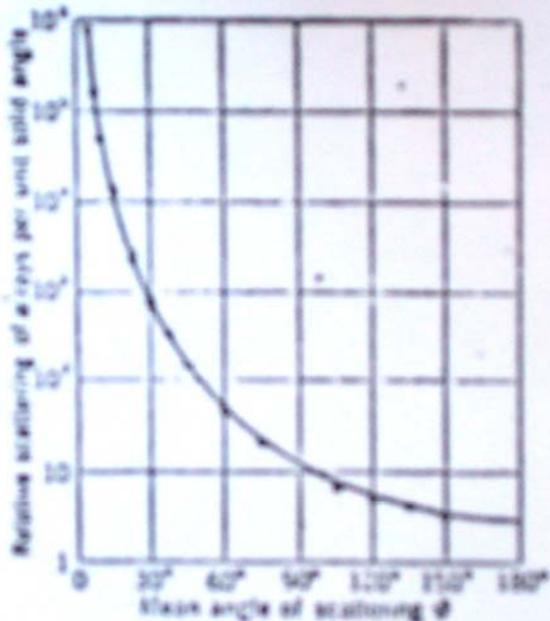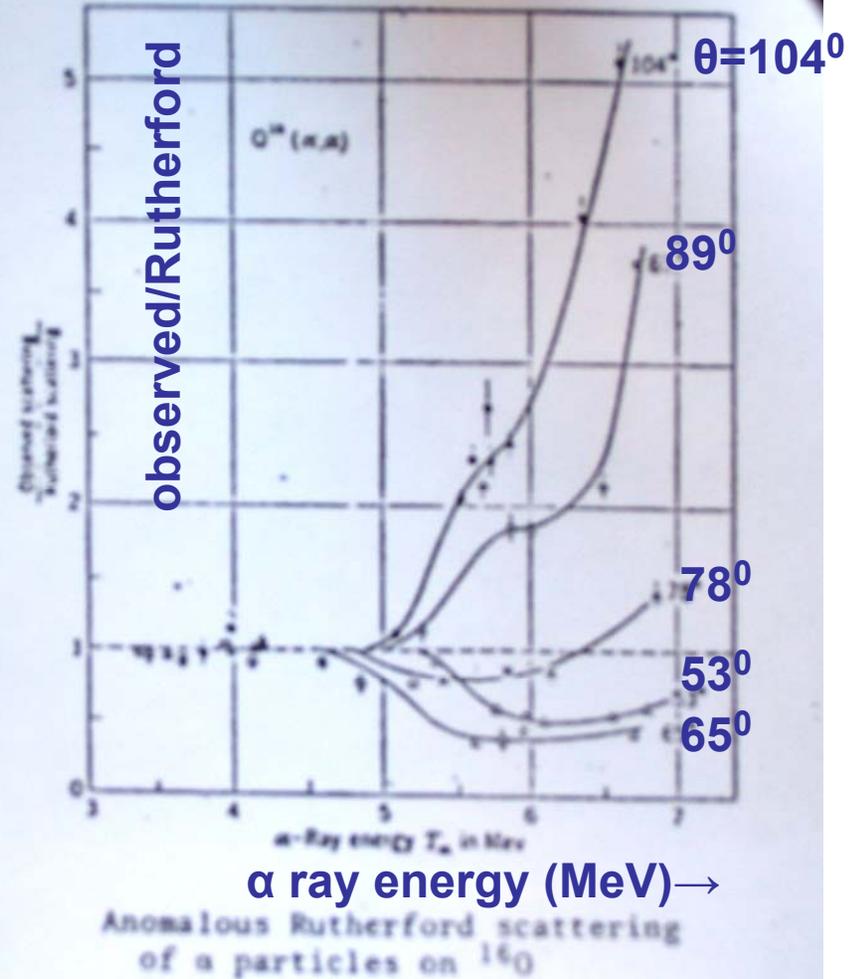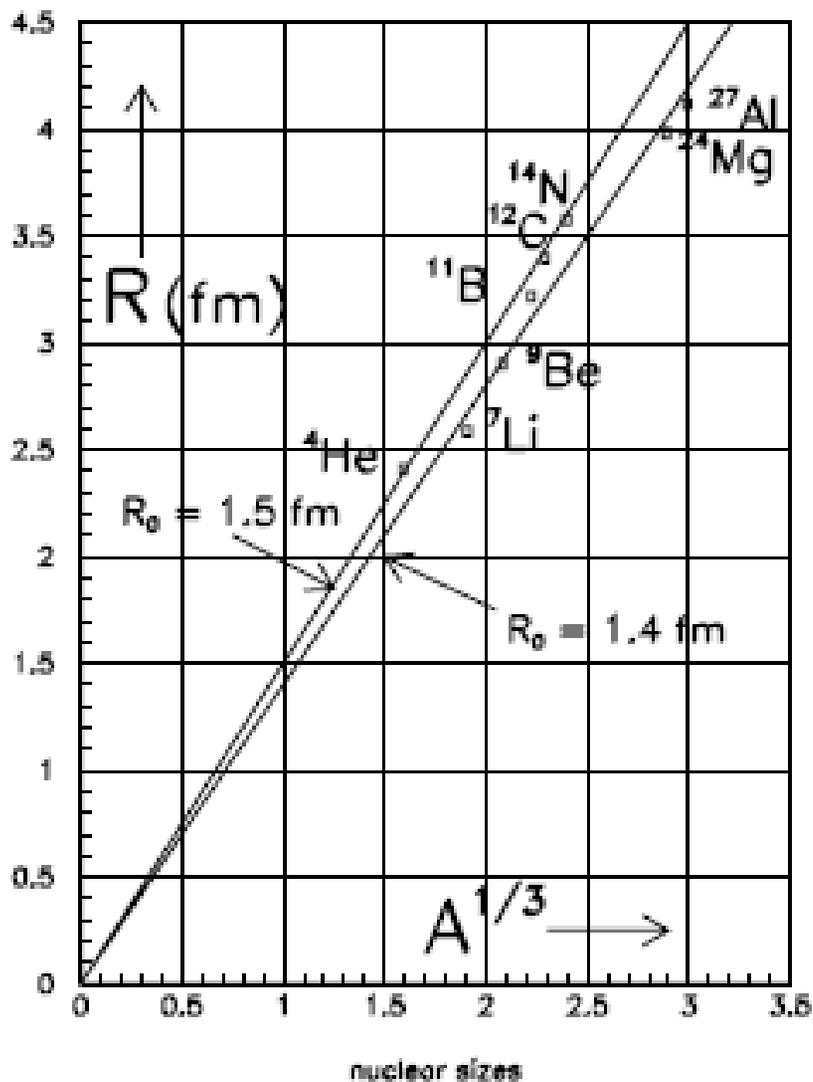
Fig. 3.3 Differential cross section for the angular scattering of α rays by a thin foil of gold. The vertical scale represents the relative number of α rays scattered into a constant element of solid angle at the mean scattering angle θ which are shown on the horizontal scale.

observed/Rutherford

θ=104⁰

89⁰

78⁰

53⁰

65⁰

α ray energy (MeV)→

Anomalous Rutherford scattering of α particles on ¹⁶O

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The results of such measurements for a number of nuclei is shown in the figure.

Plotted here is the nuclear radius as a function of the cube-root of the mass number A.

To a very good approximation one finds

$$R = R_0 A^{1/3}$$

with $R_0 = 1.4 \text{ to } 1.5 \text{ fm}$

<u>Nuclear size by electron-nuclear scattering.</u>

Another method of measuring nuclear sizes is based on electron-nuclear scattering. Since electrons do not experience the strong nuclear force, they interact only electromagnetically. In this type of experiment one therefore measures the **electric charge distributions** of the nuclei.

The most comprehensive studies of nuclear sizes by electron scattering were done in the 1950s at the Stanford Linear Accelerator Center (SLAC) by the group of Richard Hofstadter. Electron beams of from about 200 to 450 MeV were incident on nuclear targets from helium to bismuth.

According to particle-wave duality, electrons of momentum p correspond to matter waves of wavelength $\lambda = h/p$ (**de Broglie wavelength of the electron**). Therefore a few hundred MeV are required for the de Broglie wavelength to be of the order of magnitude of nuclear sizes.
Under such conditions one observes diffraction patterns which can be Interpreted as being produced by electrons scattered by spherically symmetric charge distributions. The radial dependence is described by an empirical formula of the following form:

$$\rho(r) = \rho_0 \left[1 + \exp\left(\frac{r-c}{a} \right) \right]^{-1}$$
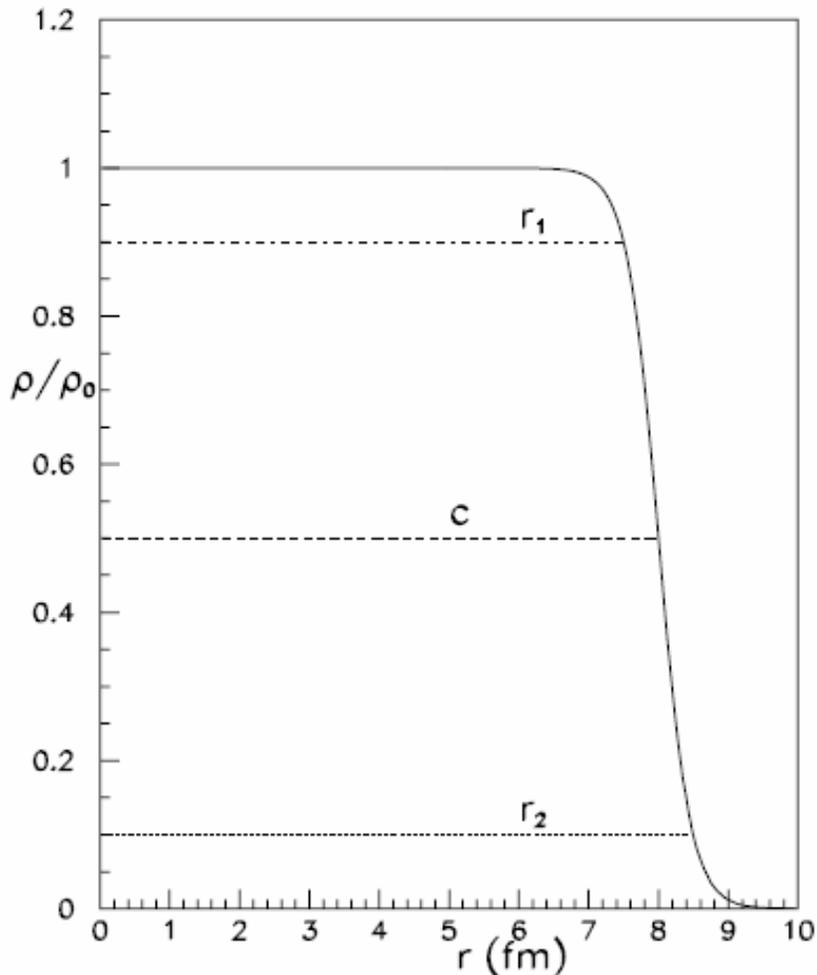
a and c are parameters, adjusted to give the best fit to the measured charge distribution; ρ_0 is a normalization factor: the charge density, integrated over the infinite volume, must give the charge of the nucleus.

The physical meaning of the parameters is elucidated in the figure: c is the width at half height, and

$$a = 4(r_2 - r_1)\ln 3$$

i.e. a characterizes the thickness of the diffuse surface layer of the nucleus.

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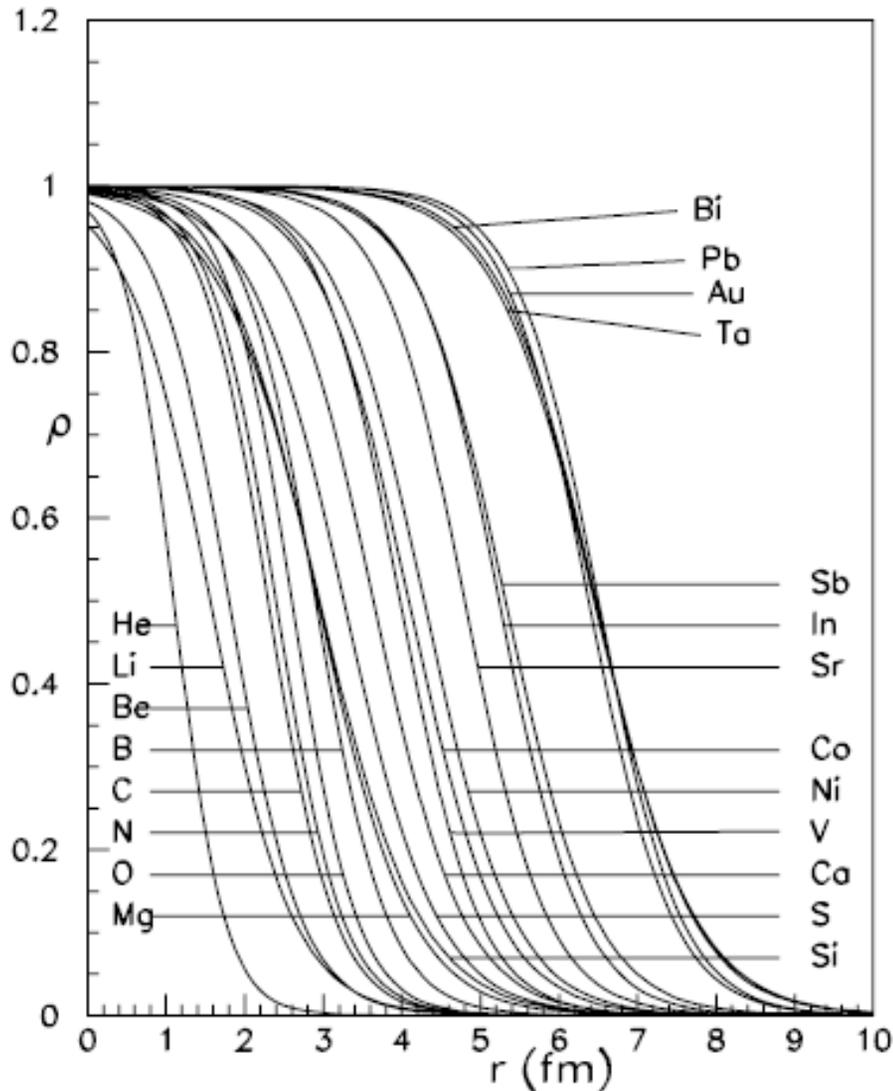
It is evident from the figure that the heavier nuclei have greater radii.

The tabulated values of the parameter c multiplied by the cube-root of A are seen to be constant within 10%.

This is striking evidence for the nuclear matter behaving like an incompressible fluid.

This property is taken as the basis of the liquid drop model of nuclei which will lead us to the semi-empirical mass formula.

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Nuclide Symbol A	c fm	$cA^{-1/3}$
He 4	1.10	0.70
Li 6	1.56	0.86
Be 9	1.80	0.87
B 11	2.00	0.90
C 12	2.30	1.00
N 14	2.40	0.99
O 16	2.60	1.03
Mg 24	2.93	1.01
Si 28	2.95	0.97
S 32	3.26	1.03

Nuclide Symbol A	c fm	$cA^{-1/3}$
Ca 40	3.64	1.06
V 51	3.98	1.07
Ni 58	4.28	1.10
Co 59	4.09	1.06
Sr 88	4.80	1.08
In 115	5.24	1.08
Sb 132	5.32	1.07
Ta 181	6.45	1.14
Au 197	6.38	1.10
Pb 208	6.50	1.10
Bi 209	6.47	1.09

1.6) Nuclear binding energy

The binding energy of a composite system is the energy needed to separate all its constituents.

The *nuclear* binding energy is the energy needed to separate all *nucleons*.

If energy is spent, then it must – by the law of conservation of energy – appear in another form.

In the case of the nucleus it is the energy associated with mass:
the mass of the nucleus is less than the sum of the masses of its constituent nucleons.

The binding energy $B(A,Z)$ of a nucleus of atomic number Z and mass number A is defined by

$$B(A,Z) = Zm_p c^2 + Nm_n c^2 - M'(A,Z)c^2$$

where the mass of the nucleus is denoted by $M'(A,Z)$

It is more convenient to use the mass of the atom and not the mass of the nucleus. Therefore we have
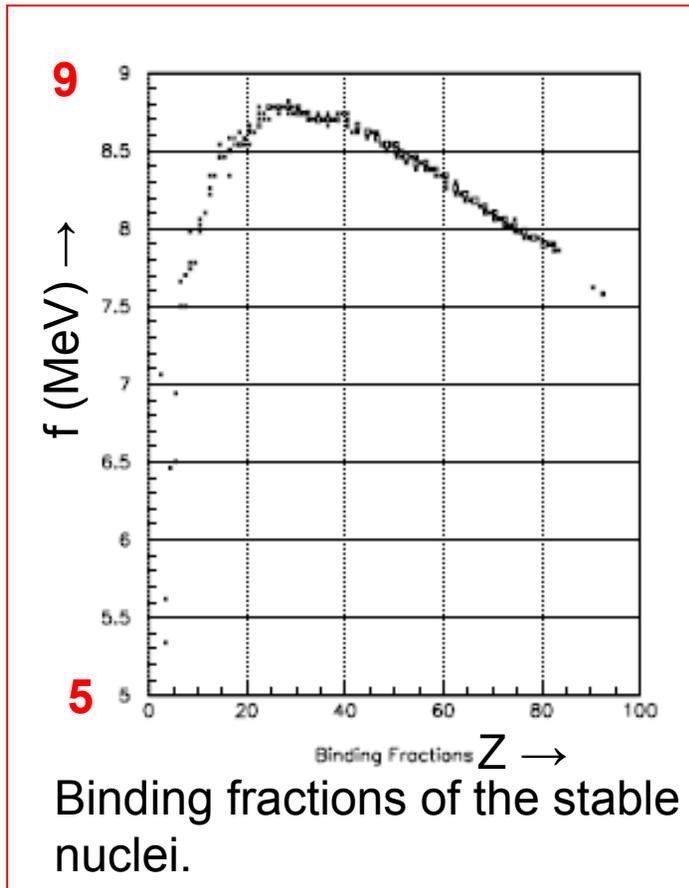
$$M(A,Z) = M'(A,Z) + Zm_e$$

(neglecting the atomic binding energy), hence

$$B(A,Z) = Zm_H c^2 + Nm_n c^2 - M(A,Z)c^2$$

and one also defines the **binding fraction** or **BE per nucleon**:

$$f = B(A,Z)/A$$

All binding fractions lie between 5 and 9 MeV

Ignoring the light elements with Z<15, the limits are even tighter:

$$\boxed{7.5 < f < 8.8 \, \text{MeV}}$$

There is a **systematic trend**:

below Z = 26 (iron) the binding fraction increases,
then it drops systematically down to about 7.5 MeV.

Thus

$$B = fA \sim 10A \, \text{MeV}$$

whereas

$$M\left(A,Z\right)c^2 \sim 1000A \, \text{MeV}$$

i.e.

$$\boxed{BE\big/Mc^2 \sim 1\%}$$

9

5

f (MeV) →

Binding Fractions $Z \rightarrow$

Binding fractions of the stable nuclei.

In nuclear reactions (e.g. explosions) only the **BE** can be released, and not the entire rest energy ("Mass Energy")

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2.) The Liquid Drop Model of the Nucleus

The nuclear volume is to a good approximation proportional to its mass. That implies that the density of nuclear matter is approximately constant. This is also the property of liquids. Therefore the simplest picture of a nucleus is that of a liquid drop.

Semi-empirical Mass Formula.

To evaporate m grams of a liquid one needs const x m Joules of heat. The reason for this is the short range of the intermolecular forces in a liquid.

Nuclear forces are also of short range. One therefore expects that the BE is approximately proportional to A, which is empirically found.

Let us ask what *other effects* there may be which can affect the binding energy.

• Guided by the liquid drop model we realize that there must be *surface tension*: the nucleons at the surface experience an attractive nuclear force only from the interior of the nucleus.

• The *Coulomb force* between the protons is a repulsive force, weakening the nuclear bond; this is of long range ($\sim 1/r^2$).

• Less obvious are two more effects:
an *asymmetry energy* and a *pairing interaction*

Surface energy:

nucleons at the surface have fewer nearest neighbors, so we expect the surface energy to be a correction proportional to the surface area *i.e.* proportional to the square of the nuclear radius and hence to $A^{2/3}$

$$SE \propto R^2 \propto A^{2/3}$$

We expect $SE < 0$ since we have overestimated the contribution of the nucleons at the surface when we set BE proportional to A.

Thus we have arrived at a formula

$$B = a_1 A - a_2 A^{2/3} + \ldots; \quad a_{1,2} > 0$$

<u>Coulomb energy:</u>

From classical electrostatics we take the definition of the charge density:

$$\rho = \frac{\text{charge}}{\text{volume}} = \frac{Ze}{(4\pi/3)R^3}$$

and from the liquid drop model we set **ρ** = constant.

Hence the electrostatic potential at the surface of a sphere of radius $r<R$:

$$V = \frac{q}{4\pi\varepsilon_0 r} = \frac{(4\pi/3)r^3\rho}{4\pi\varepsilon_0 r} = \frac{Ze}{4\pi\varepsilon_0 R}\left(\frac{r}{R}\right)^2$$

The next layer of nuclear matter has a charge equal to

$$dq = 4\pi r^2 dr \, \rho$$

and its potential energy is

$$Vdq = \frac{(Ze)^2}{4\pi\varepsilon_0} \frac{3r^4}{R^6} dr$$

and hence the total Coulomb energy is

$$\int_0^{Ze} Vdq = \frac{(Ze)^2}{4\pi\varepsilon_0} \frac{3}{5R}$$

and since R is proportional to $A^{1/3}$ we get the following contribution to the binding energy:

$$CE = -a_3 \, Z^2 \Big/ A^{1/3}$$

where $a_3 > 0$ since the Coulomb force is **weakening** the nuclear bond.

Now, the Coulomb term should vanish at $Z = 1$; therefore we replace Z^2 by $Z(Z-1)$, so we have finally

$$CE = -a_3\, Z\left(Z-1\right)\Big/ A^{\frac{1}{3}}$$

Asymmetry energy:

There is experimental evidence for discrete nuclear energy levels, similar to the discrete atomic energy levels. We will discuss this later in connection with radioactivity.

There is also an important law, the ***Pauli exclusion principle***, according to which **no two identical spin-1/2 particles**, called ***fermions***, **can occupy the same quantum mechanical state.**

Protons and neutrons are spin-1/2 fermions, so they obey the exclusion principle.

Therefore there can be only one or two protons in one energy level; if there are two protons, then their spins are oriented in opposite directions, and therefore they are in different q.m. states. Similarly for neutrons.

E

protons   neutrons
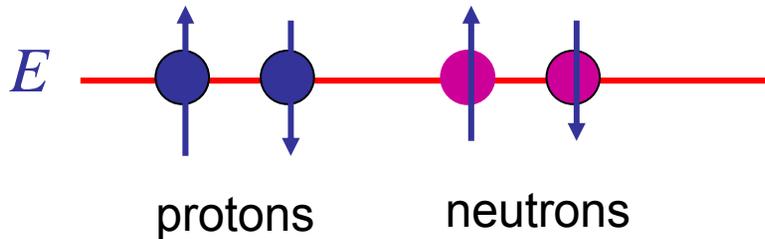
Now consider the isobars with $A = 4n$:

   if $N = Z$, then there are n filled energy levels.

   Now consider the case $N \neq Z$, and consider only
   the energy levels above the completely filled ones.
   They can be occupied in the following configurations:

4 p   3 p + 1 n   2 p + 2 n   1 p + 3 n   4 n
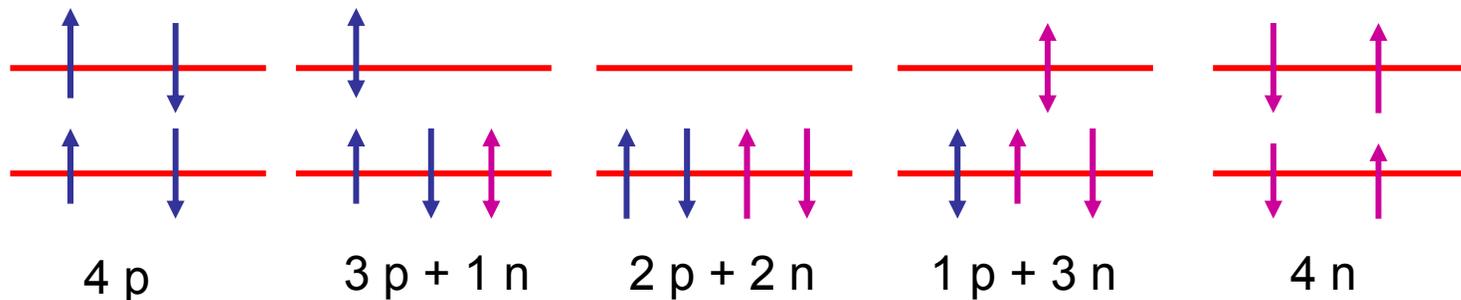
we conclude from this picture that the nucleus with $N = Z$ is more tightly bound than those with $N \neq Z$

Thus we put in the semi-empirical mass formula an asymmetry term

$$AE \propto f\left(N - Z\right)\Delta E$$

where ΔE is a typical separation between the energy levels.

In order to get an idea of the functional form of $f\left(N - Z\right)$, consider a pair of **mirror nucle**i, for example nitrogen-15 and oxygen-15

$^{15}_{7}N$ $^{15}_{8}O$

8n+7p   7n+8p
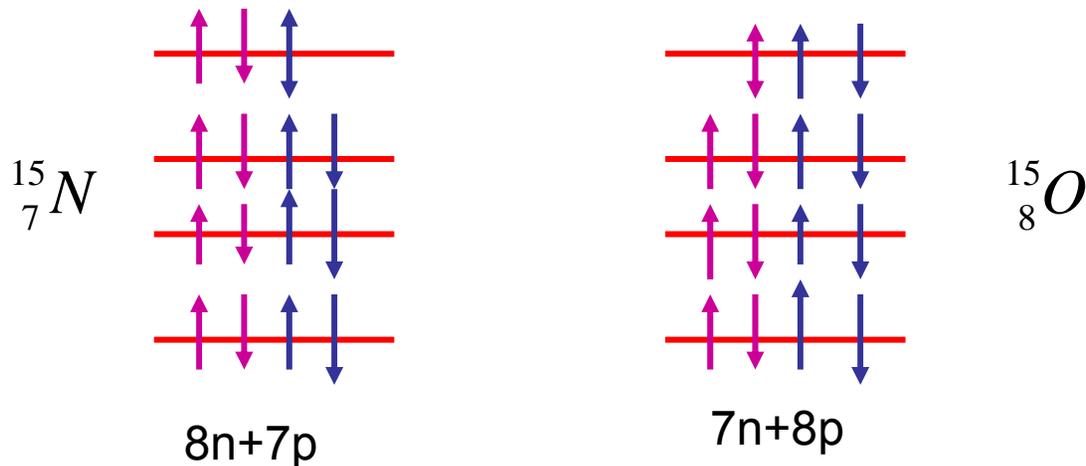
Assuming the strong nuclear force between protons and neutrons to be equal (charge independence of the nuclear force), the binding energy of mirror nuclei is the same. Any difference arises from the different Coulomb forces. This is actually observed. Therefore we conclude that

$$f(N-Z) = f(Z-N)$$

i.e we have an even function of (N – Z), and the simplest form is

$$f(N-Z) = (Z-N)^2$$

The separation between the energy levels gets smaller for heavier nuclei. A reasonable assumption is

$$\Delta E \sim 1/A$$

thus finally

$$\boxed{AE = -a_4(N-Z)^2/A}$$

with $a_4 > 0$.

Pairing interaction:

From the stability rules we know that

$$BE\left(even-even\right) > BE\left(even-odd\right) > BE\left(odd-odd\right)$$

Therefore include a term

$$\delta = \pm a_5 \Big/ A^{3/4}$$

where the "+" is for even-even nuclides, the "-" is for odd-odd nuclides, and for odd-A nuclides (i.e. even-odd and odd-even) $\delta = 0$.

The power of ¾ is empirical, and is not used by all authors: the famous "Theoretical Nuclear Physics" by Blatt and Weisskopf uses A^{-1}.

Thus finally we get the semi-empirical mass formula:

$$\boxed{B(A,Z) = a_1 A - a_2 A^{2/3} - a_3 \frac{Z\left(Z-1\right)}{A^{1/3}} - a_4 \frac{\left(A-2Z\right)^2}{A} + \delta}$$

The coefficients a_i are found by fits to nuclear binding energies.

Different authors give slightly different values, but representative values are the following:

$$a_1 = 14\,\text{MeV}$$
$$a_2 = 13\,\text{MeV}$$
$$a_3 = 0.6\,\text{MeV}$$
$$a_4 = 19\,\text{MeV}$$
$$a_5 \simeq 30\,\text{MeV}$$

The SEMF gives only the main trend.
It must not be used to calculate binding energies.
It fails for the lightest nuclides.

Useful applications of the SEMF:

(i) Mass parabolas for isobars (beta-unstable nuclides)
     I will show a large number of examples

(ii) Limit of stability against alpha-decay and spontaneous fission.

These applications will be discussed in the lectures on radioactivity.