Selected Topics in Physics

a lecture course for 1st year students

by W.B. von Schlippe

Spring Semester 2007

Lecture 10

Radioactive Decay of Nuclei

Some naturally occurring substances have the property to emit radiation.

This was discovered by *Antoine Henri Becquerel* in 1896.

The evidence was a blackening of photographic emulsion wrapped in black cardboard on which Becquerel had placed a piece of a mineral (a uranium salt).

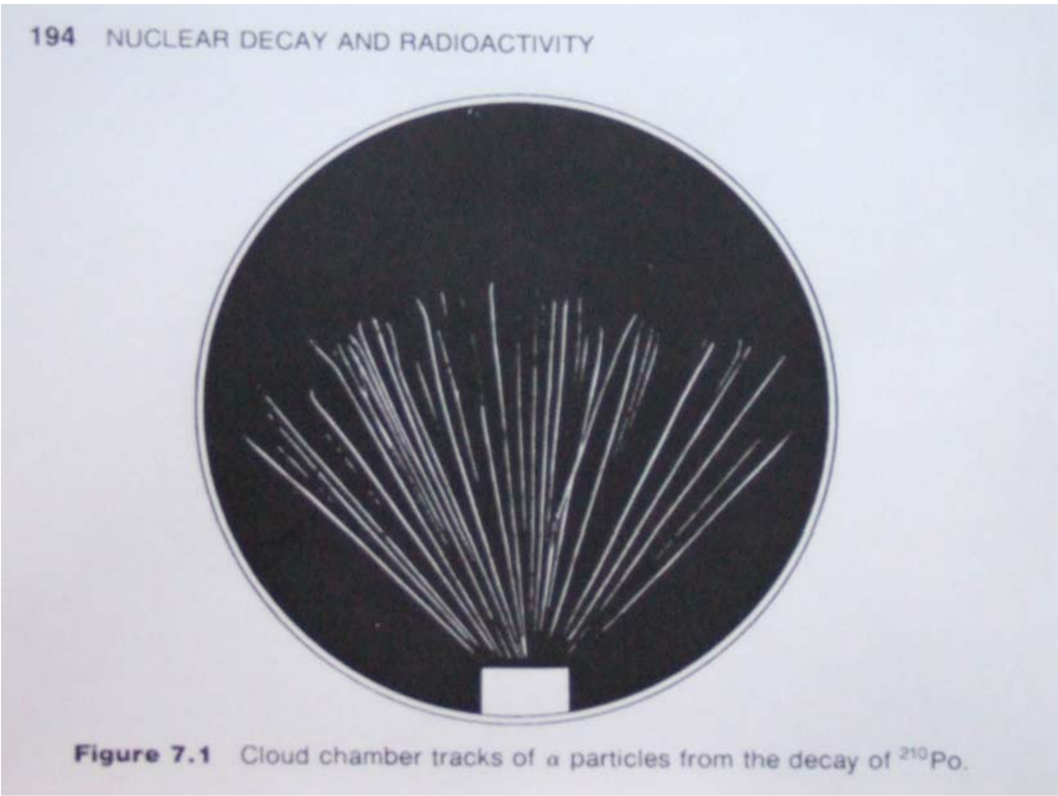
The conditions of the experiment excluded the possibility of exposure of the emulsion to light.

There had to be an unknown kind of radiation coming from the mineral.

This phenomenon was called ***radioactivity***.

Further study of radioactivity showed that there were ***three kinds of radiation***: ***α***, ***β*** and ***γ*** rays.

α rays: they have a fairly well defined range in air as demonstrated in a photograph of tracks in a Wilson cloud chamber.



from K.S. Krane, *Introductory Nuclear Physics*

α rays are deflected by electric and magnetic fields. From the sense of deflection one concludes that they have positive charge.

Their ***specific charge*** e/m was shown to be about 4000 times less than the specific electron charge.

β* rays** are more strongly deflected by electric and magnetic fields and in the direction opposite to that of α rays, so they are ***negatively charged; their range is ***not*** well defined.

***γ* rays** are not deflected; they are like X rays but of shorter wavelength; emission of γ rays does not lead to transmutations of atoms.

α rays were identified by Rutherford as consisting of helium ions or, as we say today, as the nuclei of helium.

β rays were shown to be electrons.

Radioactivity is a process that involves individual atoms; it is not a collective phenomenon of an assembly of atoms.

In the basic radioactive process a nuclear decay takes place in which an atom emits an *α* particle or an electron (*i.e*. a *β* particle) or a photon.

Examples of *α* decay:

$$^{226}_{88}Ra \to\ ^{222}_{86}Rn + {}^{4}_{2}\alpha + Q$$

$$^{238}_{92}U \to\ ^{234}_{90}Th + {}^{4}_{2}\alpha + Q$$

where Q is the kinetic energy released in the process.

Examples of *β* decay:

$$^{1}_{0}n \to {}^{1}_{1}p + {}^{0}_{-1}e^{-} + \bar{\nu} + 0.782\,\mathrm{MeV}; \quad 10.2\ \text{minutes}$$

$$^{3}_{1}H \to {}^{3}_{2}He + {}^{0}_{-1}e^{-} + \bar{\nu} + 0.019\,\mathrm{MeV}; \qquad 12.3\text{years}$$

$$^{10}_{4}Be \to {}^{10}_{5}B + {}^{0}_{-1}e^{-} + \bar{\nu} + 0.556\mathrm{MeV}; \qquad 1.5\times10^{6}\ \text{years}$$

$$^{64}_{29}Cu \to {}^{64}_{30}Zn + {}^{0}_{-1}e^{-} + \bar{\nu} + 0.58\,\mathrm{MeV},\ 39\%$$

$$^{64}_{29}Cu \to {}^{64}_{28}Ni + {}^{0}_{1}e^{+} + \nu + 0.653\mathrm{MeV},$$

$$^{64}_{29}Cu + {}^{0}_{-1}e^{-} \to {}^{64}_{28}Ni + \nu + 1.675\mathrm{MeV}, \qquad 61\%$$

In the last example it is shown that there are three kinds of β decay: β^-, β^+ and ***electron capture*** (EC).

The α decay scheme is consistent with the picture of equal ranges of the α ray tracks.

Indeed, the methods of relativistic kinematics of particle collisions (Lecture 7) can be applied to particle decays. Thus we can find the momentum of the ***daughter nucleus*** Y assuming that the ***mother nucleus*** X is initially at rest:

$$p^* = \frac{1}{2M_X}\left\{\left[M_X - \left(M_Y - m_\alpha\right)^2\right]\left[M_X - \left(M_Y + m_\alpha\right)^2\right]\right\}^{1/2}$$

and the α particle has the same momentum ***(Exercise!)***.

Thus the momenta of all α particles are equal. Travelling through air they lose energy in small portions ionizing molecules along their tracks. Thus they spend their initial kinetic energy at roughly equal distances traveled.

Beta decay is a ***1-to-3 body decay***. Here the kinematics is more involved. The result of the kinematical analysis is that the beta-decay electron can have momenta from zero to a maximum which is defined by the masses of the particles involved.

Actually, since the neutrino mass is negligibly small if not zero, the formula for the maximum momentum of the β particle is like the formula for the momentum of the α particle given above but with m_α replaced by m_β:

$$p_{e\max} = \frac{1}{2M_X}\left\{\left[M_X - \left(M_Y - m_\beta\right)^2\right]\left[M_X - \left(M_Y + m_\beta\right)^2\right]\right\}^{1/2}$$

Note that in this formula and in the previous one the speed of light was set equal to 1.

The Law of Radioactive Decay

In an ensemble of radioactive atoms every atom decays independently of all other atoms at an instant in time that cannot be predicted.

Thus radioactivity is a ***random process*** governed by the law of probability.

If there is a sample of N atoms, then the number dN of atoms decaying in an interval of time dt is proportional to N. Considering that the number of atoms in the sample is ***decreasing***, we have therefore

$$dN = -\lambda N \, dt$$

This is the basic law of radioactive decay. $\boldsymbol{\lambda}$ is the ***decay constant***

Integrating we get

$$N = N_0 \, e^{-\lambda t}$$

where N_0 is the number of atoms in the sample at time $t = 0$.

The time it takes for half of the sample to decay is the ***half-life***:

$$N\left(t_{1/2}\right) \equiv N_0 \exp\left(-\lambda t_{1/2}\right) = \frac{1}{2} N_0$$

hence

$$\boxed{t_{1/2} = \frac{\ln 2}{\lambda}}$$

The ***mean life*** is defined by

$$\tau = \int_{N_0}^{0} t\, dN \Bigg/ \int_{N_0}^{0} dN = \int_0^{\infty} t\, de^{-\lambda t} \Bigg/ \int_0^{\infty} de^{-\lambda t} = \frac{1}{\lambda}$$

Counting the number of atoms in a sample is not practical. One therefore defines a related quantity, the ***activity*** of the sample:

The activity of sample of a radioactive substance is the number of disintegrations per second:

$$A = -dN/dt$$

hence

$$A = \lambda N = \lambda N_0 e^{-\lambda t}$$

or if we denote the activity at time $t = 0$ by A_0, then

$$A = A_0 e^{-\lambda t}$$

The SI unit of activity is the ***Becquerel***, denoted **Bq**.

1 Bq = 1 disintegration per second

An obsolete unit, the Curie, is still widely used: it is the activity of 1 gram of radium (approximately).

1 Ci = 3.7×10^{10} Bq

The ***specific activity*** is defined for samples of **<u>pure</u>** radioisotopes; it is denoted by a and defined as the activity per unit of mass of a pure radioisotope.

Consider a radioactive substance X that disintegrates into a stable substance Y and assume that initially there were no atoms of Y in the sample, then at time t we have

$$N_X = N_{X0} e^{-\lambda t}$$

$$N_Y = N_{X0} \left(1 - e^{-\lambda t}\right)$$

The total number of atoms is constant: it is equal to the initial number of atoms of substance X.

Now assume that the daughter product is also unstable. Then we have:

$$dN_1 = -\lambda_1 N_1\, dt$$
$$dN_2 = \lambda_1 N_1\, dt - \lambda_2 N_2\, dt$$

The first one of these equations is solved simply, and we have done it already. The second equation can be solved by the following ***ansatz***:

$$N_2 = Ae^{-\lambda_1 t} + Be^{-\lambda_2 t}$$

If at t = 0 there were no atoms of substance 2, then

$$A + B = 0$$

and after substitution and a few lines of straight forward calculation we get

$$N_2 = N_{10} \frac{\lambda_1}{\lambda_2 - \lambda_1} \left(e^{-\lambda_1 t} - e^{-\lambda_2 t} \right)$$

If the daughter product is stable, then $\lambda_2 = 0$ and we regain the previous formula for the build-up of a stable daughter product.

The activity of the daughter product comes only from its decays, hence

$$A_2 = \lambda_2 N_2 = N_{10} \frac{\lambda_1 \lambda_2}{\lambda_2 - \lambda_1} \left(e^{-\lambda_1 t} - e^{-\lambda_2 t} \right)$$

The decay of substance 2 leads to a build-up of its decay product (***grand-daughter***); thus the complete set of equations is

$$\begin{aligned} dN_1 &= -\lambda_1 N_1 \, dt \\ dN_2 &= \lambda_1 N_1 \, dt - \lambda_2 N_2 \, dt \\ dN_3 &= \lambda_2 N_2 \, dt \end{aligned}$$

assuming that substance 3 is stable. We can see immediately that …

$$dN_1 + dN_2 + dN_3 = d\left(N_1 + N_2 + N_3\right) = 0$$

hence

$$\boxed{N_1 + N_2 + N_3 = \text{constant}}$$

Assume that initially we had

$$N_1 = N_0, \quad N_2 = N_3 = 0$$

then

$$\boxed{N_3 = N_0 \frac{\lambda_1 \lambda_2}{\lambda_2 - \lambda_1}\left[\frac{1}{\lambda_1}\left(1 - e^{-\lambda_1 t}\right) - \frac{1}{\lambda_2}\left(1 - e^{-\lambda_2 t}\right)\right]}$$

(***Exercise!***)

The ***decay chains*** found in nature or artificially produced consist of many more generations. One can continue the analysis of the build-up of all intermediate unstable isotopes and of the final stable isotopes.

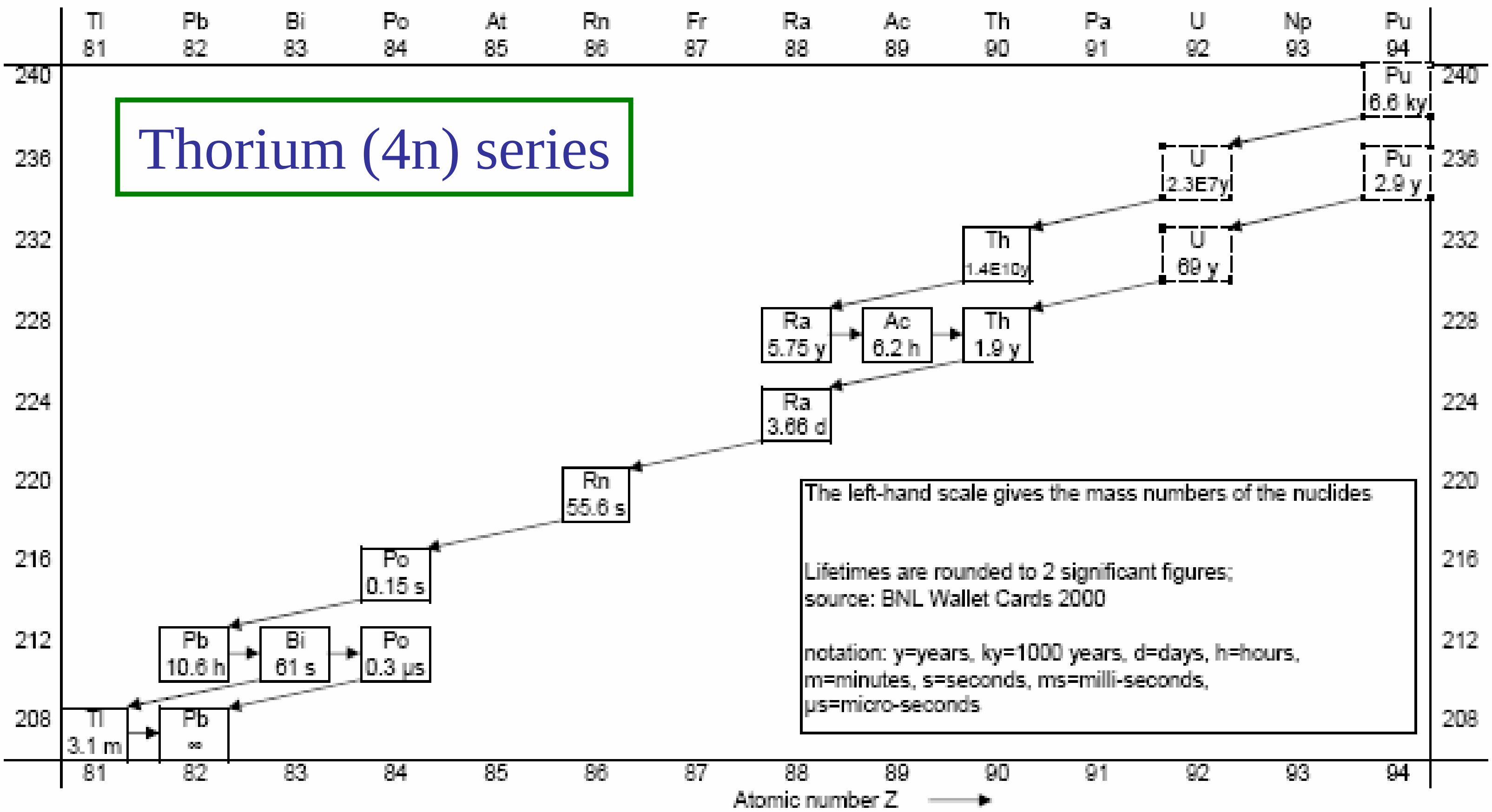
Such analyses form the basis of methods of dating rock, meteorites and dead organic matter and is therefore of great practical interest.

There are 4 series of radioactive substances. They are substances whose atomic numbers are A = 4n, 4(n+1), 4(n+2) and 4(n+3), where n is an integer.

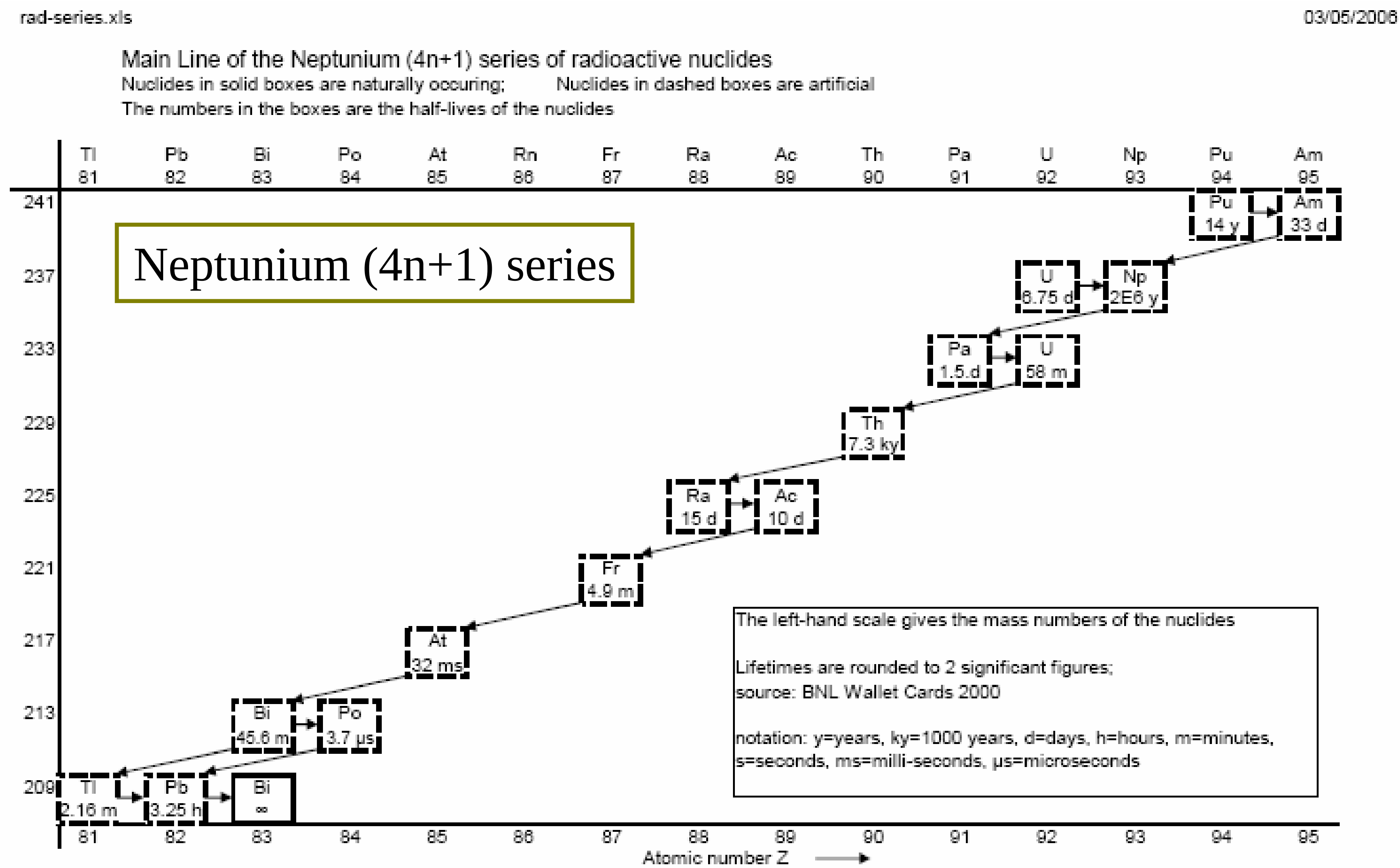
The four series are shown in the following slides.

Thorium (4n) series

Main Line of the Thorium (4n) series of radioactive nuclides
Nuclides in solid boxes are naturally occuring; Nuclides in dashed boxes are artificial
The numbers in the boxes are the half-lives of the nuclides

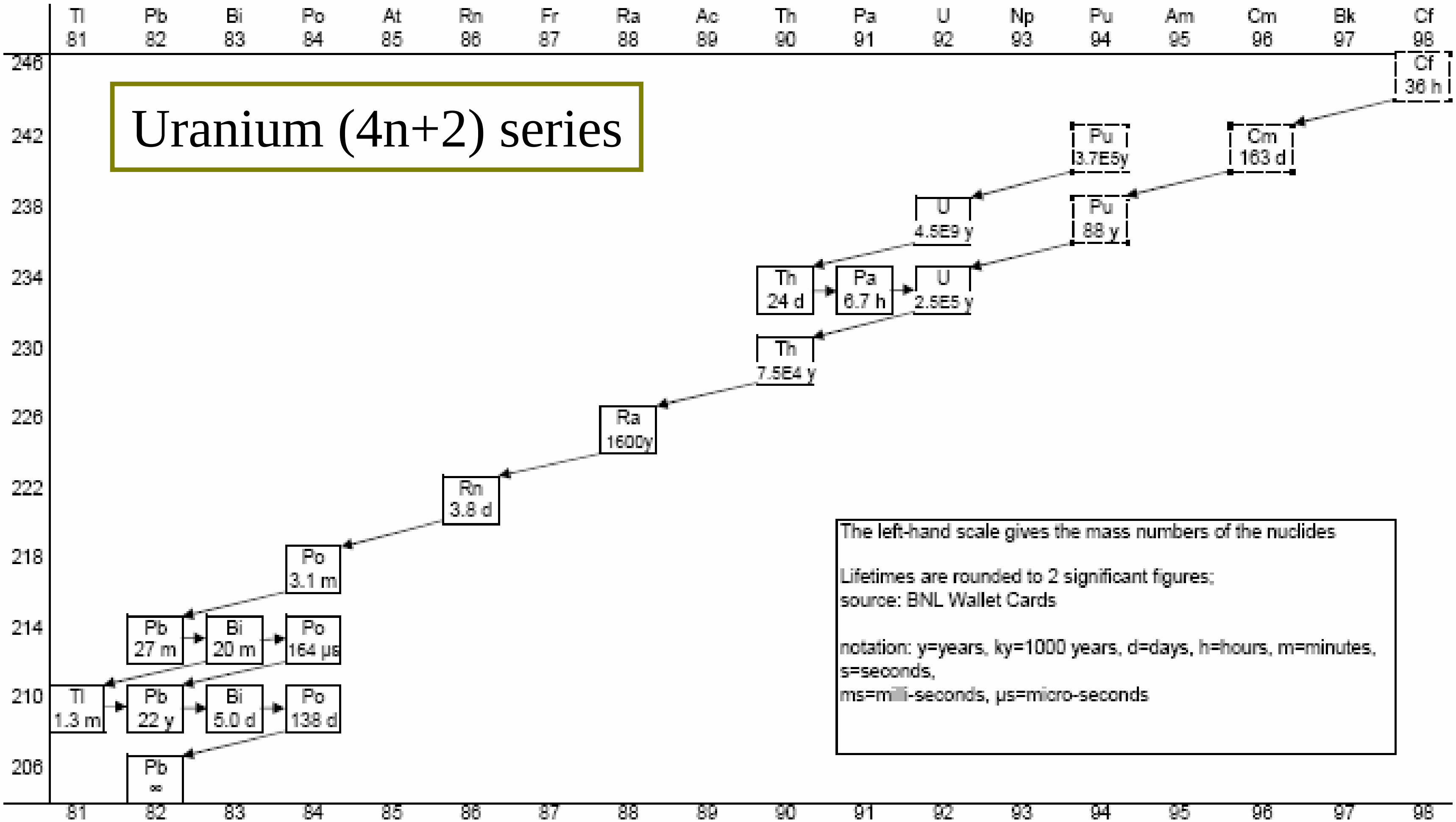


Neptunium (4n+1) series

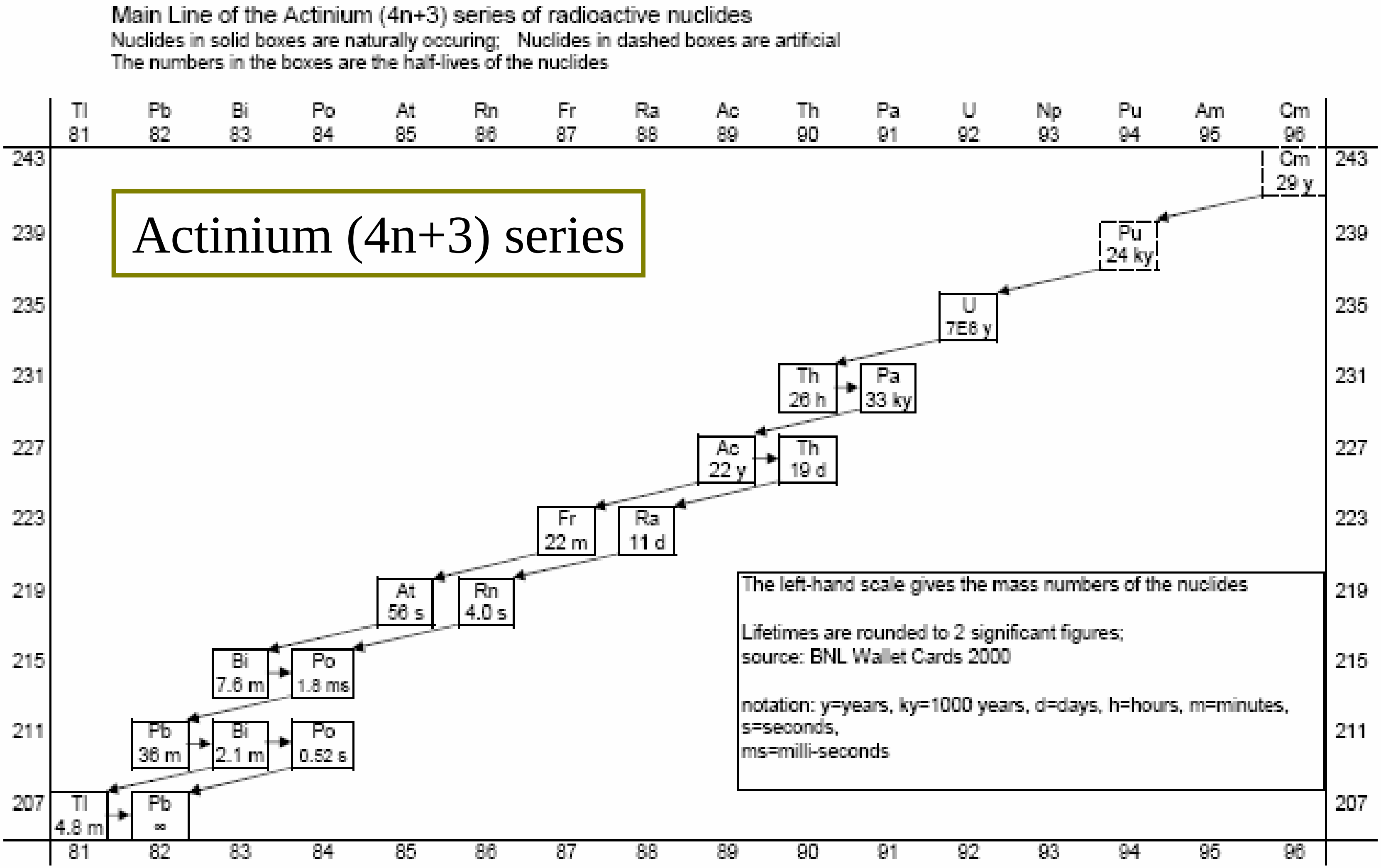


Main Line of the Uranium (4n+2) series of radioactive nuclides
Nuclides in solid boxes are naturally occuring; Nuclides in dashed boxes are artificial
The numbers in the boxes are the half-lives of the nuclides

Uranium (4n+2) series



Actinium (4n+3) series



Beta decay crisis: violation of energy conservation?

In the early years of studying nuclear beta decay it was ***wrongly*** assumed that the beta decay reaction had the form of

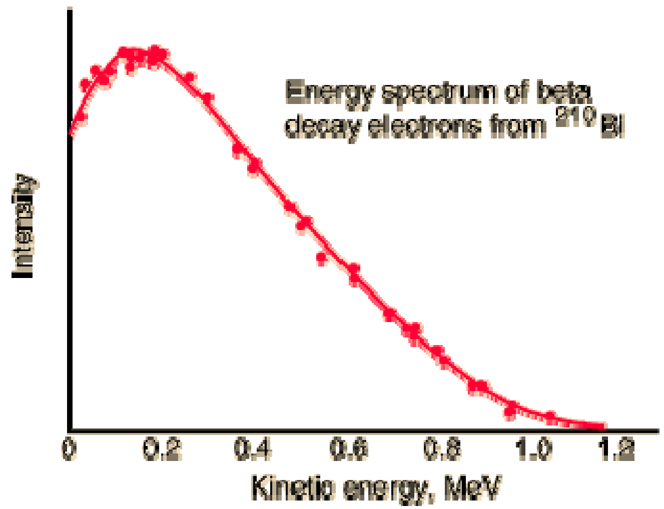
$$X(A,Z) \rightarrow Y(A,Z+1) + e^-$$

Then the electron should have a momentum given by

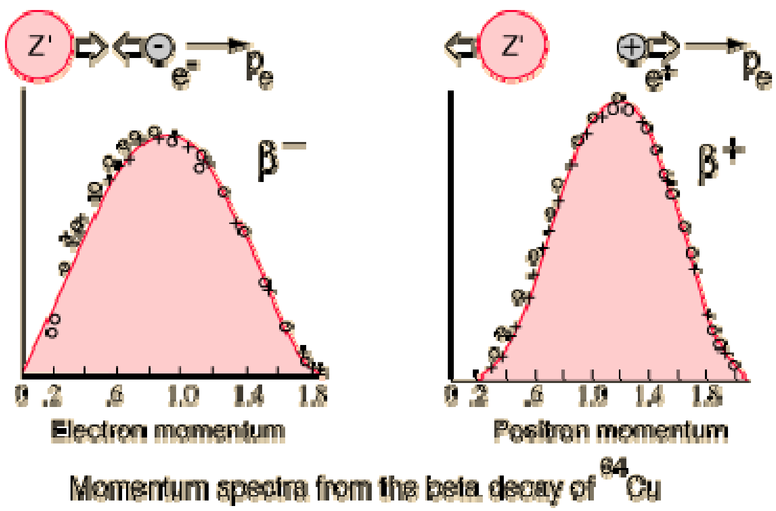
$$p = \frac{1}{2M_X}\sqrt{\left[M_X{}^2 - (m_Y - m_e)^2\right]\left[M^2 - (m_Y + m_e)^2\right]}$$

The particular value of p for a specific beta decay is not important. Important is that the momentum, and hence the energy of the electron should have a fixed value defined by the masses of the three particles involved in the decay. This is not observed as can be seen on the following examples.

Experimental Beta Decay Spectra:



From G.J. Neary, Proc. Phys. Soc. (London), **A175**, 71 (1940).



From J.R. Reitz, Phys. Rev. **77**, 50 (1950).

From these and many more examples it is seen that the electrons from beta decay have a continuum of energies ranging from zero to some maximum value.

This discrepancy between theory and experiment was the ***beta decay crisis***.

One way out was suggested by **Niels Bohr**:

a possible violation of the conservation of energy in nuclear processes.

Another way out was proposed by **Wolfgang Pauli**: the existence of a neutral particle that was not "seen" in beta decay. Pauli called it "***neutron***". When soon after that the neutron was discovered by Chadwick, Enrico Fermi gave Pauli's neutral particle the name ***neutrino***: "little neutron" in Italian; that name stuck.

Niels Bohr

Wolfgang Pauli

James Chadwick

Enrico Fermi

Thus according to Pauli the beta decay reaction must be written as

$$X(A,Z) \rightarrow Y(A,Z+1) + e^- + \nu$$

but after several more years it was understood that there existed not only a neutrino but also an ***antineutrino***. The modern assignment is such that in beta decay the ***antineutrino*** is emitted together with the ***electron*** and the ***neutrino*** together with the ***positron***.

Thus the reaction equations are written in the general form of

$$X(A,Z) \rightarrow Y(A,Z+1) + e^- + \overline{\nu} \quad \text{in } \beta^- \text{ decay}$$

$$X(A,Z) \rightarrow Y(A,Z-1) + e^+ + \nu \quad \text{in } \beta^+ \text{ decay}$$

Basic for all nuclear beta decays is the neutron decay:

$$^1_0n \rightarrow \, ^1_1p + \, ^0_{-1}e^- + \overline{\nu} + Q$$

The third kind of beta decay is electron capture (***EC***),

$$X(A,Z) + e^{-} \rightarrow Y(A,Z-1) + \nu \quad EC$$

here an ***atomic electron is captured*** by the nucleus usually from the **K** shell, *i.e.* the shell closest to the nucleus and therefore with the greatest probability of the electron spending some time in the nucleus.

Electron capture like β^+ decay changes a nucleus of atomic number Z into a nucleus of atomic number Z-1; it is therefore a process ***competing*** with β^+ decay.

Conditions for β decay

The cause of the instability that leads to β decay is an ***excess of energy***: if there is a way of getting rid of this excess energy, then the decay will take place.

Therefore consider the ***energy balance*** in β decay.

β $^{-}$ decay:

The energy balance equation reads

$$M'(A,Z)c^2 = M'(A,Z+1)c^2 + m_e c^2 + Q$$

where the M' are ***nuclear masses*** and Q is the energy released.

Obviously, the condition for the reaction to go is

$$\boxed{Q > 0}$$

If we change from nuclear masses to atomic masses, then we must add Z electron masses to the left-hand side. That gives us Z+1 electron masses on the right-hand side. Thus in terms of atomic masses the equation reads:

$$M(A,Z)c^2 = M(A,Z+1)c^2 + Q$$

or

$$\boxed{Q = M(A,Z)c^2 - M(A,Z+1)c^2 > 0}$$

i.e. for β^- decay to take place it is sufficient for the mother atom to have a mass greater than that of the daughter atom.

For β^+ decay the condition reads

$$\boxed{Q = M(A,Z)c^2 - M(A,Z-1)c^2 - 2m_e c^2 > 0}$$

and for electron capture it is identical with the condition for β^- decay.

(*Exercise*!!)

More convenient than using atomic masses in the conditions for beta decay is the use of the mass excess. The mass excess Δ is defined by

$$\Delta(A,Z) = M(A,Z) - A$$

and hence the conditions for beta decay can be written in the following form:

$$\beta^-, EC: \quad \Delta(A,Z) - \Delta(A,Z+1) > 0$$

$$\beta^+: \quad \Delta(A,Z) - \Delta(A,Z-1) > 2m_e$$

Note that one must be careful not to confuse mass excess with mass defect: they are equal in magnitude but opposite in sign.
The extensive tables of nuclides ***BNL Wallet cards***, which are accessible on the internet, use the mass excess.
We will see that the mass excess has a powerful intuitive meaning when applied to beta decay.

From the formulas of the conditions for beta decay it appears that we must compare the masses (or mass excesses) of neighboring isobars to decide whether a decay is possible.

Based on the semi-empirical mass formula we can predict what we can expect in such a comparison.

The semi-empirical mass formula which we have derived in the previous lecture gives the binding energy by the following formula:

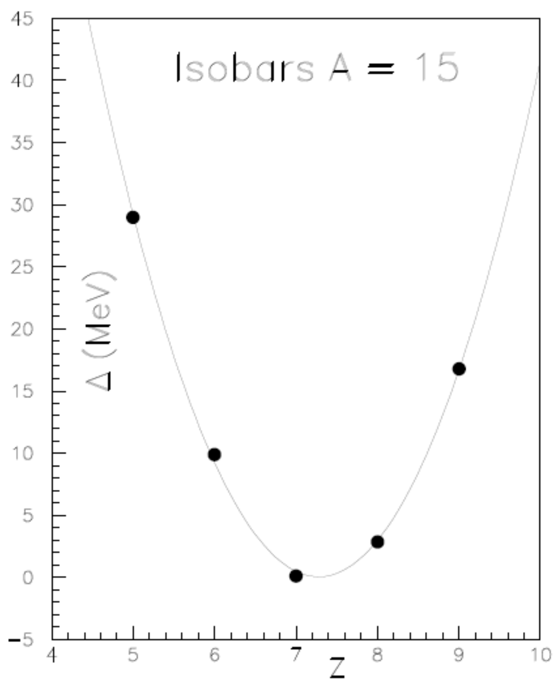
$$B(A,Z) = a_1 A - a_2 A^{2/3} - a_3 \frac{Z(Z-1)}{A^{1/3}} - a_4 \frac{(A-2Z)^2}{A} + \delta$$

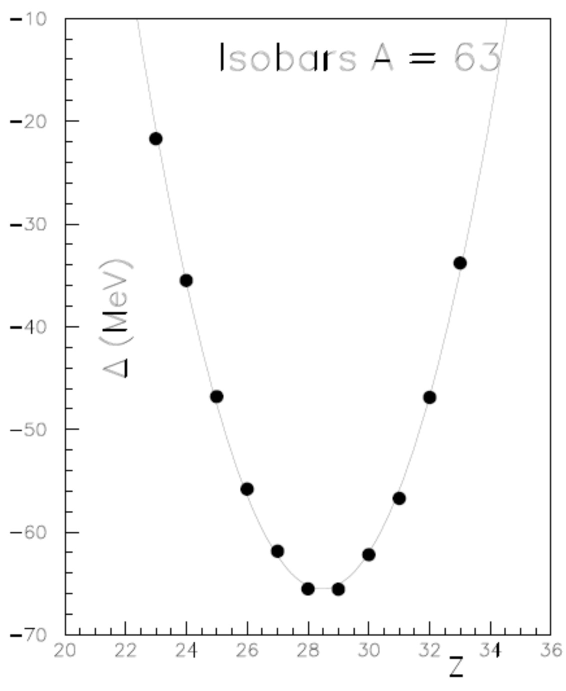
with $\delta = \pm a_5 / A^{3/4}$ or zero, where the upper sign is for even-even and the lower sign for odd-odd nuclides and the zero for odd-A nuclides.

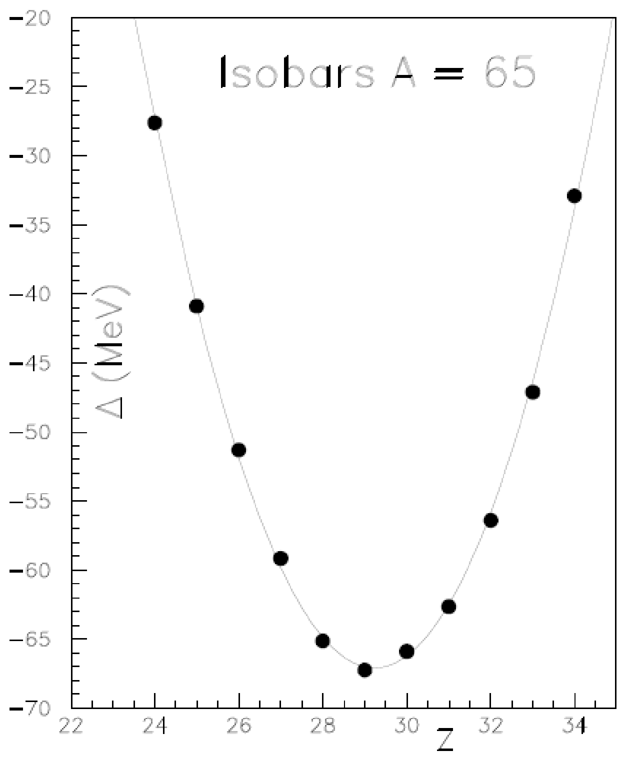
We can see that the binding energy depends quadratically on Z for fixed A. The same will be true for the mass defect. Therefore we expect the mass defects of isobars of fixed A to be of the following form:

$$\Delta = c_0 + c_1 Z + c_2 Z^2$$

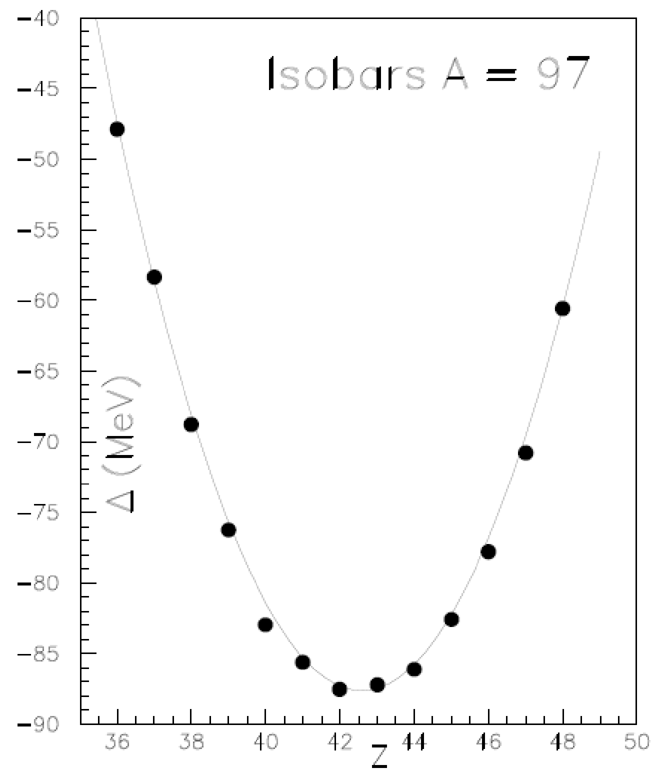
The mass excesses for some isobars are shown in the following figures.



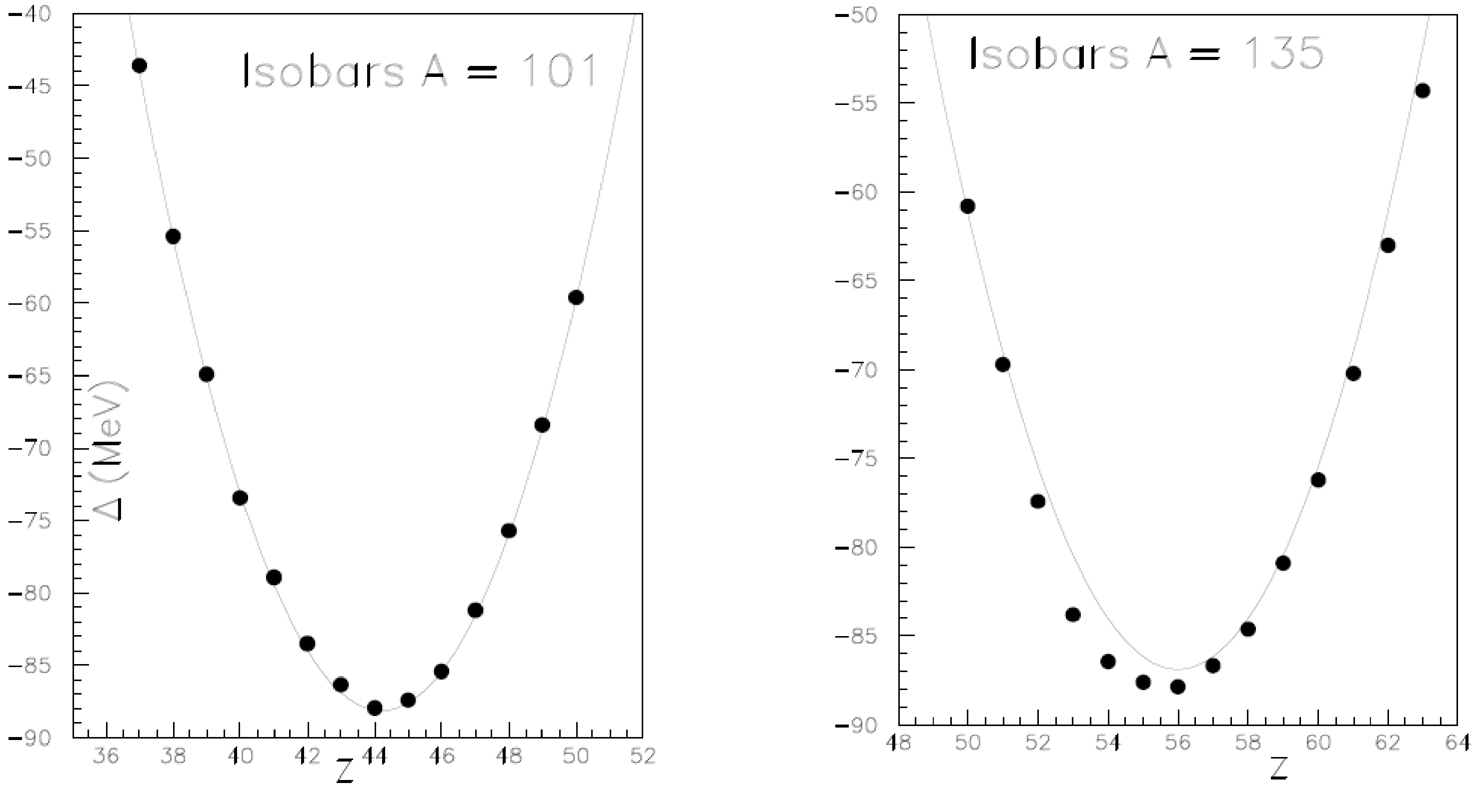




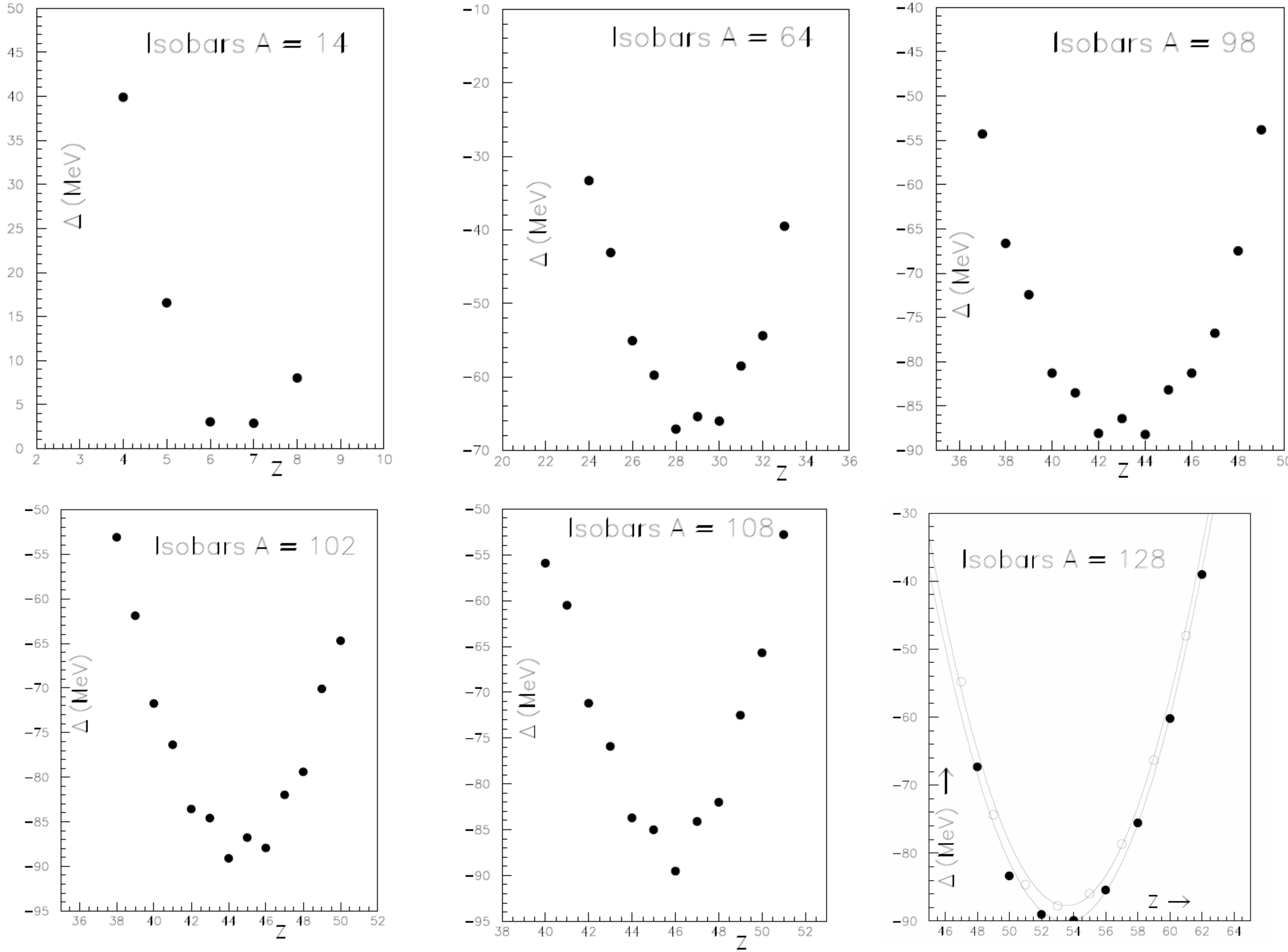
Isobars A = 65
Δ (MeV)
Z



Isobars A = 97
Δ (MeV)
Z



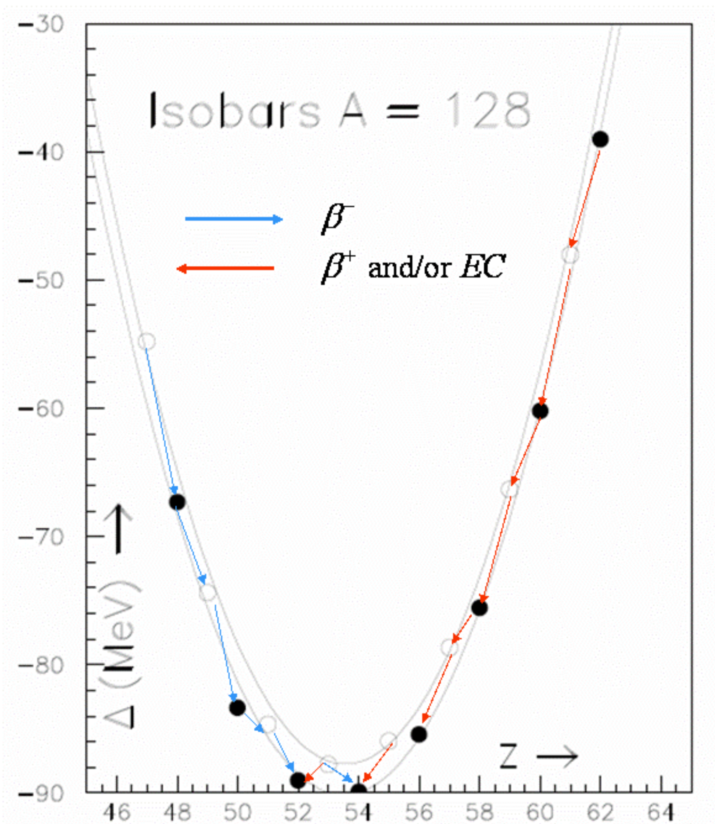
These were odd-A isobars. The even-A isobars are a bit harder to fit, so I have done only one with a fit, the others without:



Isobars A = 14
Isobars A = 64
Isobars A = 98
Isobars A = 102
Isobars A = 108
Isobars A = 128
Δ (MeV)
Z

One can clearly see in all examples, including the ones without fitted curves, that the isobars lie on ***mass parabolas***.

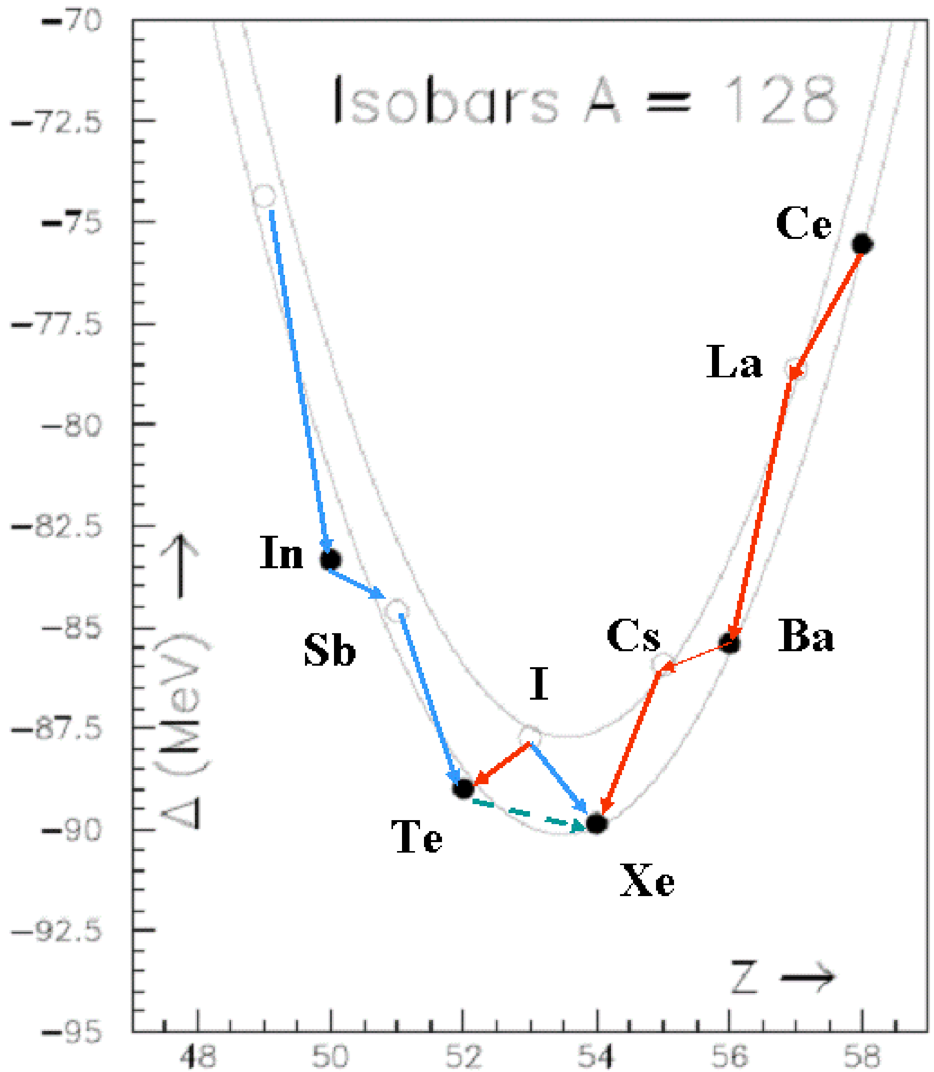
The most interesting mass parabolas are the even-A ones where there are two parabolas separated by 2δ. Let us discuss the case A = 128 in detail.

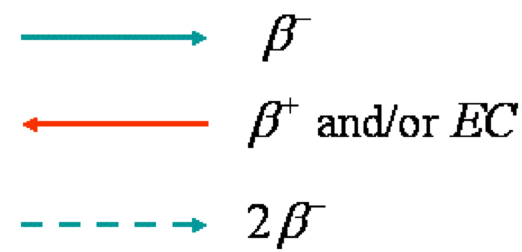


$^{128}_{53}$I:
β^- 93.1%,
ε 6.9%

Here we see that the mass excess has a very powerful intuitive meaning: the decays show up like ***sliding down a slope***!

There is also an interesting feature near the bottom of the parabola: we see directly how Iodine-128 can decay both by beta-minus decay and by electron capture (and probably not by beta-plus emission)





Double beta decay:

$\mathrm{Te} \rightarrow \mathrm{Xe} + 2\beta^- + 2\bar{\nu} + Q,$

$Q = 0.87\,\mathrm{MeV},$

$t_{1/2} = 8.8 \times 10^{18}\,\mathrm{years}$

As an interesting ***exercise(!!)*** derive the condition for double-beta decay to be energetically possible.

This is an exercise with two exclamation marks, so it is an absolute ***must*** to do it.

The study of 2β decay is one of the interesting developments of nuclear physics in recent years.

Line of Maximum beta-Stability

Empirically the line of maximum beta stability is winding and splits into two branches for most even-A nuclides.

The SEMF gives a smooth curve defined by

$$\left.\frac{\partial M}{\partial Z}\right|_{A=const} = 0$$

hence

$$Z = A \Big/ \left(1.98 + 0.015 A^{2/3}\right)$$

(Exercise!)

Electron Conversion

Electron conversion is a process in which ***nuclear excitation energy*** is transferred to an atomic electron. Since the typical nuclear excitation energy is much greater than atomic binding energy, the electron is ejected from the atom. There is no accompanying neutrino, so this is a two-body decay, and hence the conversion electrons have a discrete spectrum. In the early years of studying radioactivity such discrete spectra were confusing the correct interpretation of the phenomena, especially as the conversion spectra tend to be superimposed on continuous beta-decay spectra.

The reason for this is that frequently the daughter nucleus in a beta decay is left in an excited state, so the correct reaction equation is

$$X\left(A,Z\right)\rightarrow Y^{*}\left(A,Z+1\right)+e^{-}+\overline{\nu}$$

where Y^{*} is the excited daughter nucleus. The de-excitation can take place by gamma-ray emission or by electron conversion or both.