

High Energy Physics

Lecture 9

Deep Inelastic Scattering Scaling Violation

Deep Inelastic Scattering:

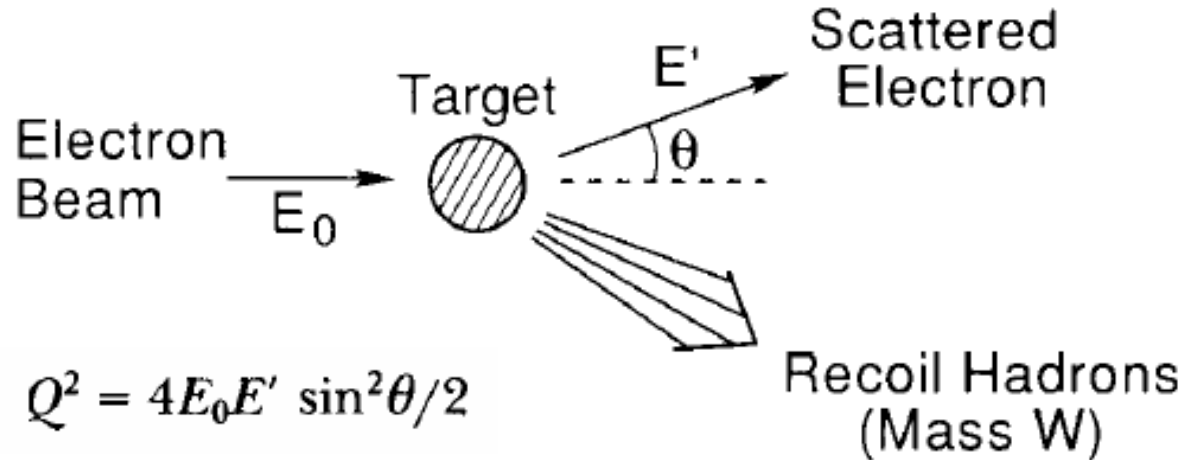
The reaction equation of DIS is written

$$e + p \rightarrow e + X$$

where X is a system of outgoing hadrons (mostly pions). Observed is only the scattered electron. The unobserved hadronic system is the *missing mass*.

The energy of the incident electron beam is accurately known. The proton is the target particle; in the SLAC experiments (and many later experiments at CERN) the target is at rest in the laboratory. This defines the LAB frame.

DIS Kinematics:



$$Q^2 = 4E_0E' \sin^2 \theta/2$$

$$E' = \frac{E_0 - \frac{(W^2 - M^2)}{2M}}{1 + \frac{2E_0}{M} \sin^2 \theta/2}$$

or, since E' and θ are measured:

$$W^2 = M^2 + 2M(E_0 - E') - 4E_0E' \sin^2 \frac{\theta}{2}$$

Note: for elastic scattering $W=M$

Feynman diagram of inelastic electron-proton scattering:

(from H.W. Kendall, "Deep Inelastic Scattering", Nobel Lecture, December 8, 1990)

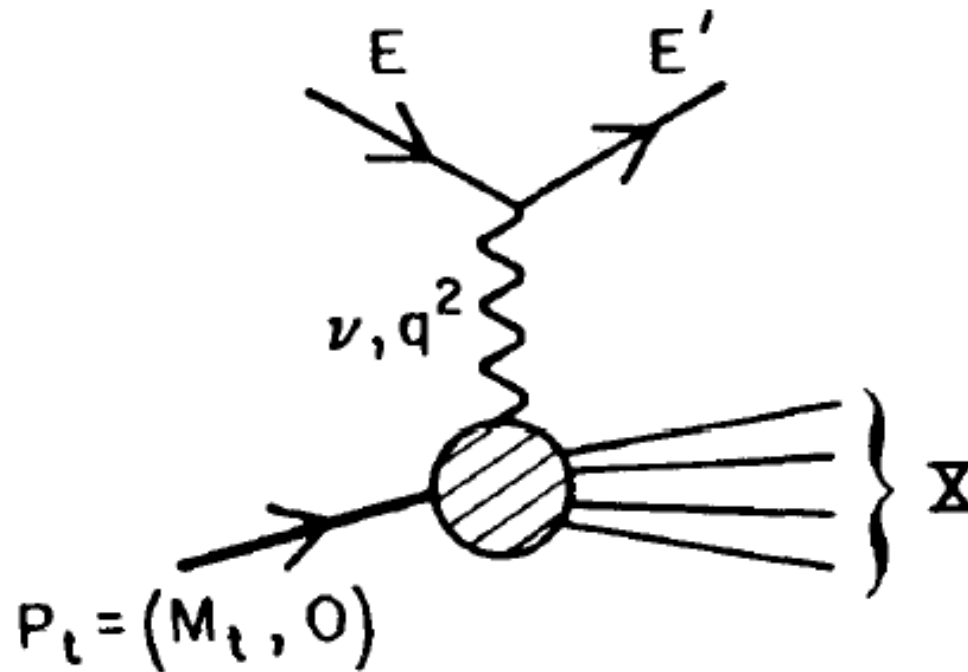


Fig. 5. Feynman diagram for inelastic electron scattering.

Differential Cross Section of Inelastic ep Scattering:

$$\frac{d\sigma}{d\Omega dE'} = \frac{\alpha^2}{4E_0^2 \sin^2 \theta/2} \cos^2 \theta/2 [W_2 + 2W_1 \tan^2 \theta/2]$$

$$W_{1,2} = W_{1,2}(Q^2, \nu) \quad \text{structure functions}$$

$$\nu = E_0 - E'$$

On general grounds the structure functions are functions of two kinematical invariants

They were expected to drop with increasing Q^2 as rapidly as the form factors

The surprising experimental result was different!

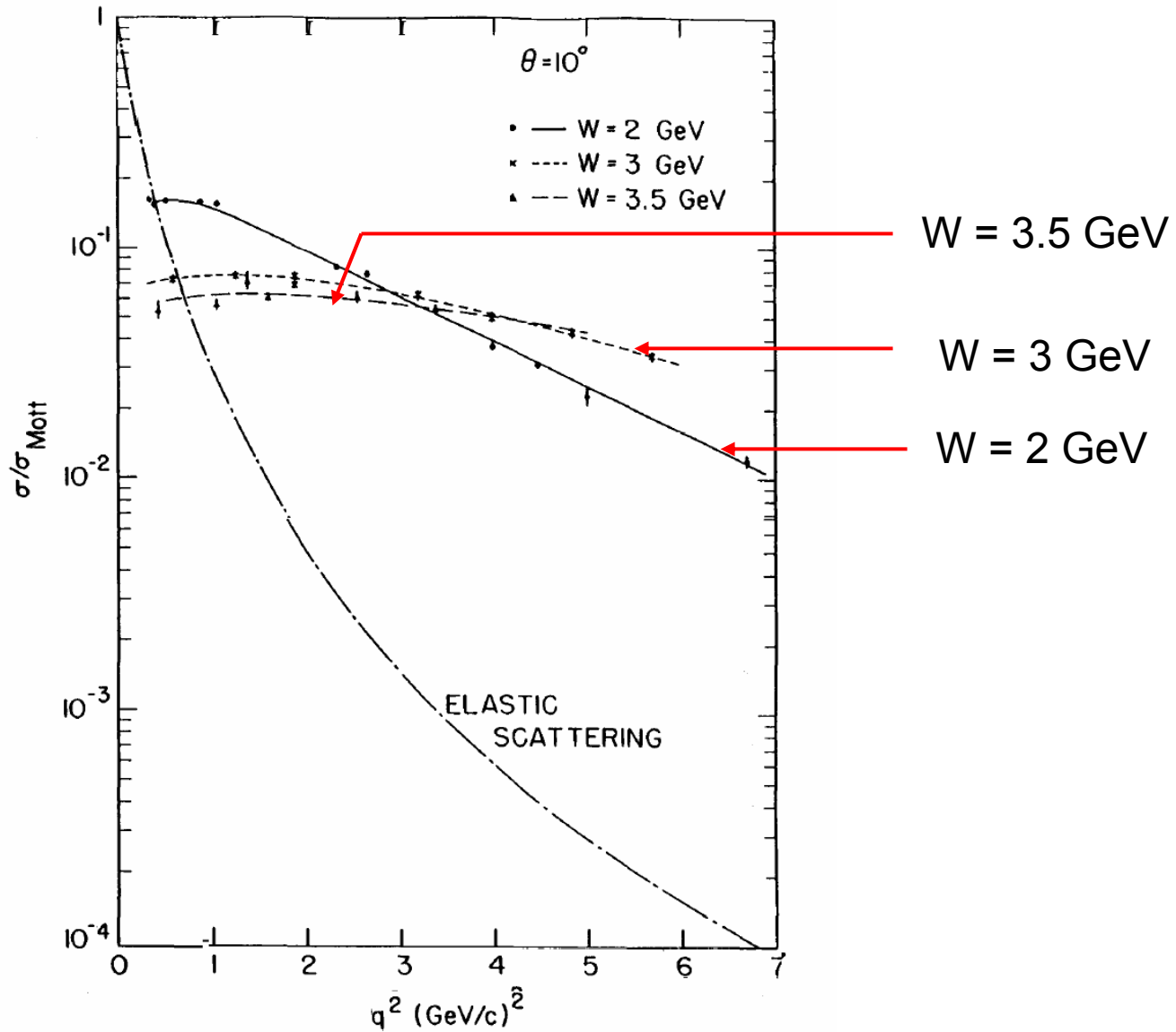
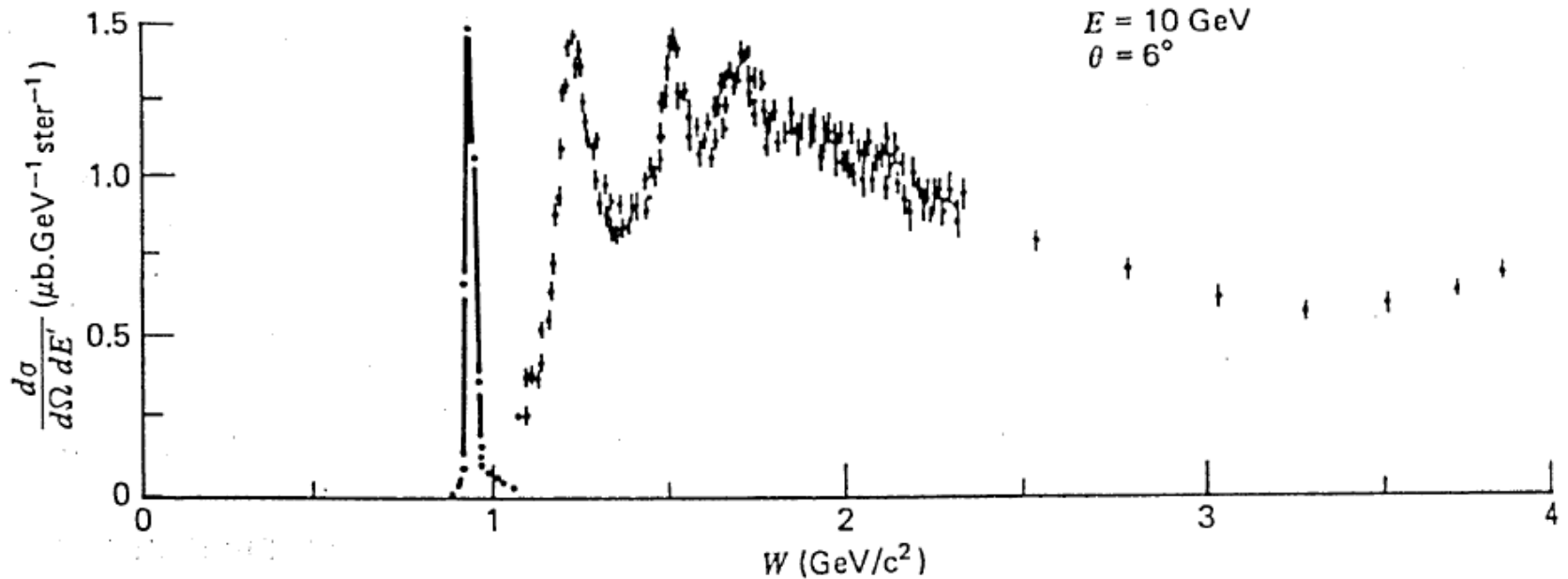


Fig. 1: $(d^2\sigma/d\Omega dE)/\sigma_{\text{Mott}}$, in GeV^{-1} , vs. q^2 for $W = 2, 3$ and 3.5 GeV. The lines drawn through the data are meant to guide the eye. Also shown is the cross section for elastic e-p scattering divided by σ_{Mott} , $(d\sigma/d\Omega)/\sigma_{\text{Mott}}$, calculated for $\theta = 10^\circ$, using the dipole form factor. The relatively slow variation with q^2 of the inelastic cross section compared with the elastic cross section is clearly shown.

The Structure of Hadrons



up to about $W = 1.8 \text{ GeV}$ there is structure corresponding to the production of **resonances** (excited nucleon states);
there is no structure above 1.8 GeV : this is the region of DIS.

Scaling:

Bjorken (1969): for

$Q^2 \rightarrow \infty$ and $\nu \rightarrow \infty$, such that $\omega = 2M\nu / Q^2$ is fixed

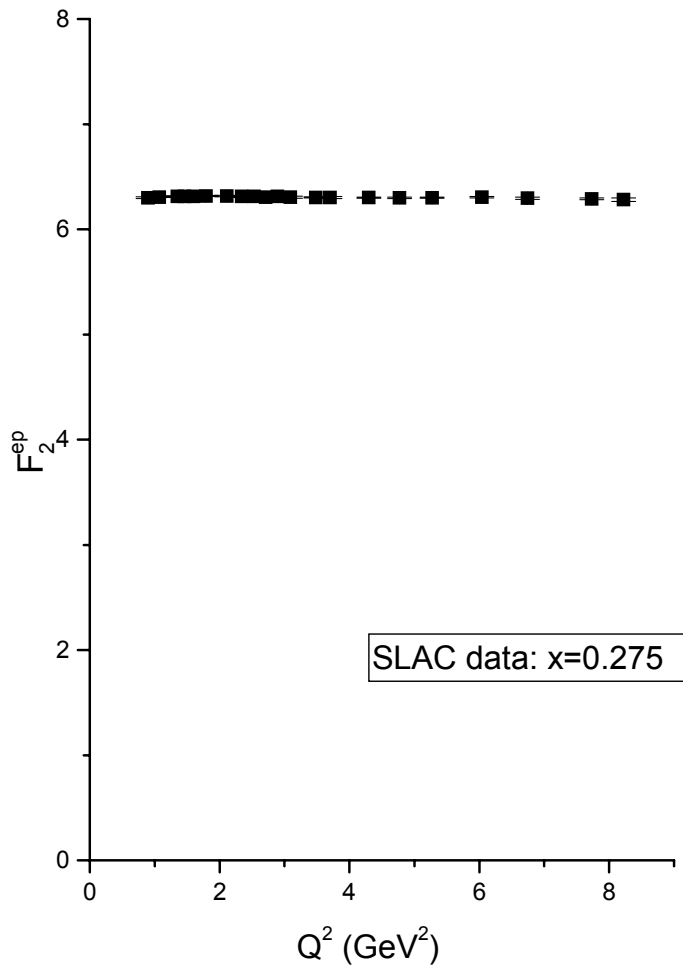
(“**Bjorken limit**”), the structure functions depend only on ω :

$$2MW_1(Q^2, \nu) \rightarrow F_1(\omega)$$

$$\nu W_2(Q^2, \nu) \rightarrow F_2(\omega)$$

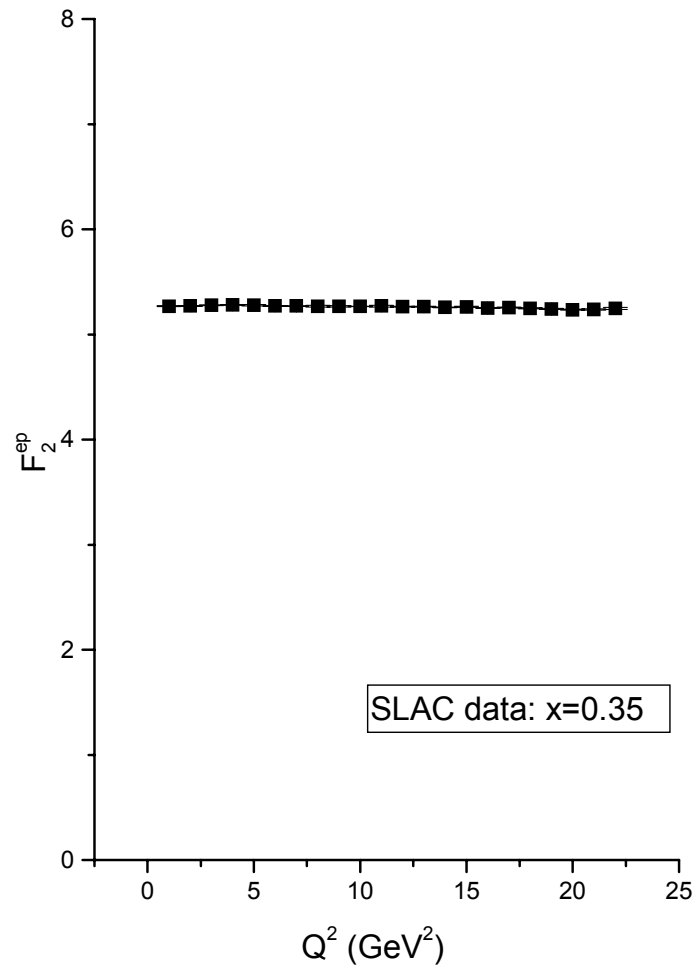
this behaviour is called **scaling**; ω is the *scaling variable*

Q^2 dependence of structure function $F_2(x, Q^2)$
SLAC data at $x=0.275$, $Q^2 = 0.9, \dots, 8.22 \text{ GeV}^2$

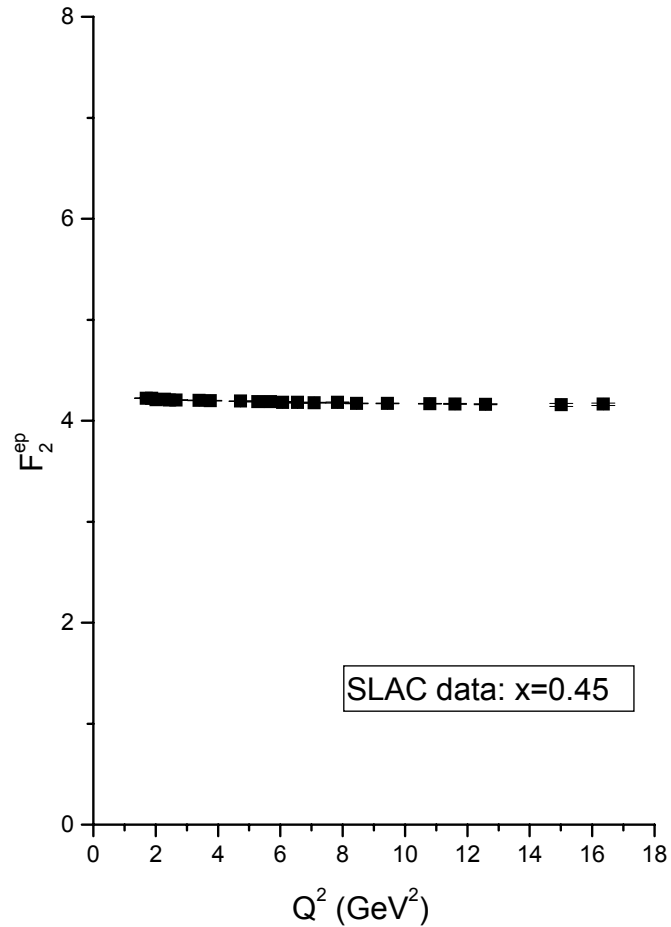


Later, a new scaling variable was defined by Feynman: $x=1/\omega$. This is now used universally and called Bjorken- x

Q^2 dependence of structure function $F_2(x, Q^2)$
SLAC data at $x=0.35$, $Q^2 = 1.1, \dots, 10.95 \text{ GeV}^2$



Q^2 dependence of structure function $F_2(x, Q^2)$
SLAC data at $x=0.45$, $Q^2 = 1.7, \dots, 16.36 \text{ GeV}^2$



Scaling found a natural explanation in the *parton model* (Feynman).

Partons are constituents of the proton (more generally of hadrons).

They are quarks and gluons:

quarks are point-like spin-1/2 fermions like the leptons,

gluons are massless spin-1 bosons: they are the carriers of the strong interaction.

Unlike leptons, quarks take part in strong as well as electromagnetic and weak interactions.

Recall:

charged leptons take part only in electromagnetic and weak interactions, neutrinos, i.e. neutral leptons, only in weak interactions.

Parton model picture of DIS:

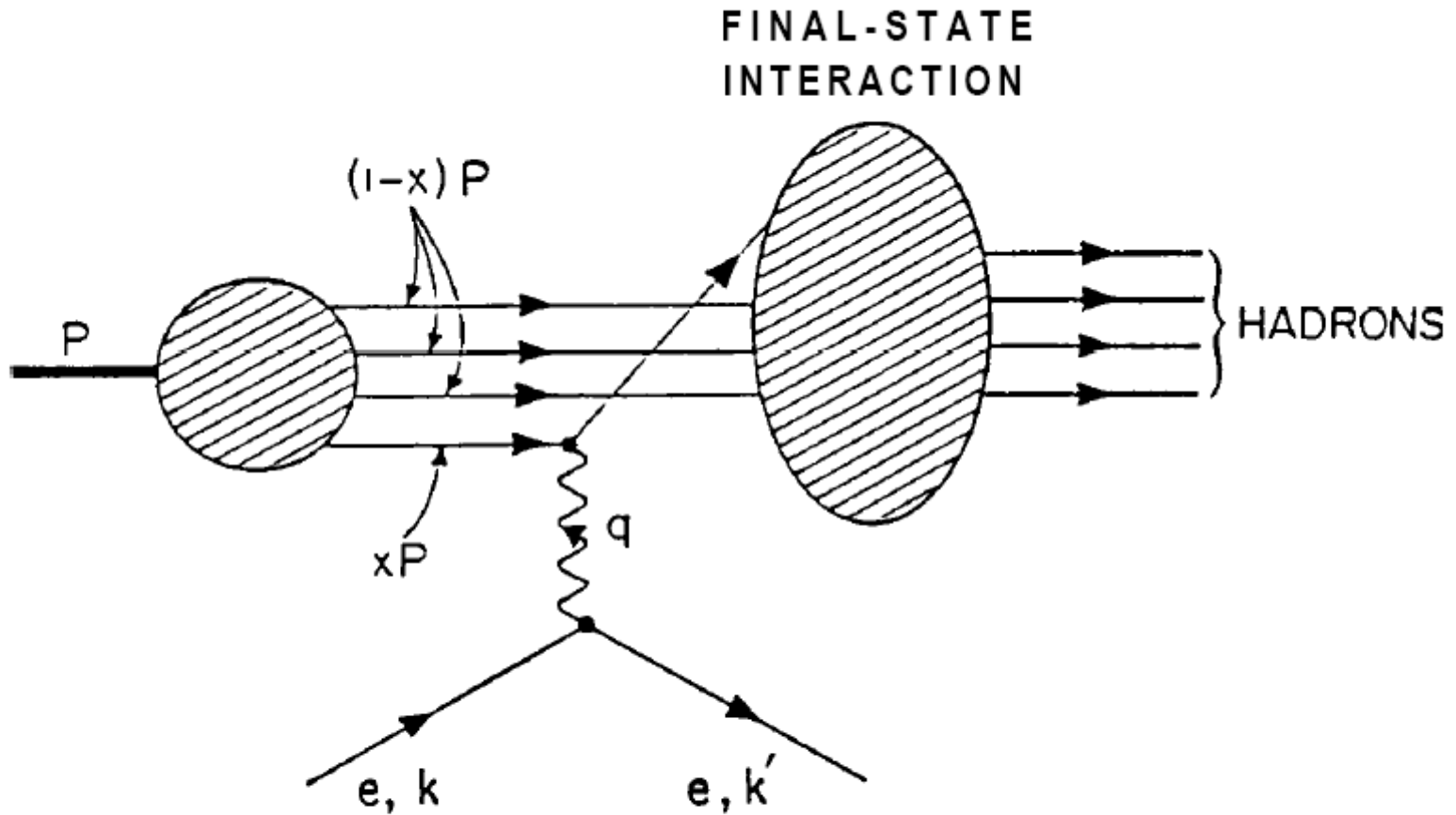


Fig. 4: A representation of inelastic electron nucleon scattering in the parton model. k and k' are the incident and final momenta of the electron. The other quantities are defined in the text.

The electron collides elastically with a parton that carries a fraction x of the proton momentum.

At high momentum (“infinite” momentum) the partons are free. Therefore the collision of one parton with the electron does not affect the other partons. This leads to scaling in x

$$x = 1/\omega = Q^2 / 2M\nu \in [0,1]$$

DIS experiments have also been done with muons and with neutrinos.

Since 1992 DIS experiments are also done on the electron-proton collider **HERA**.

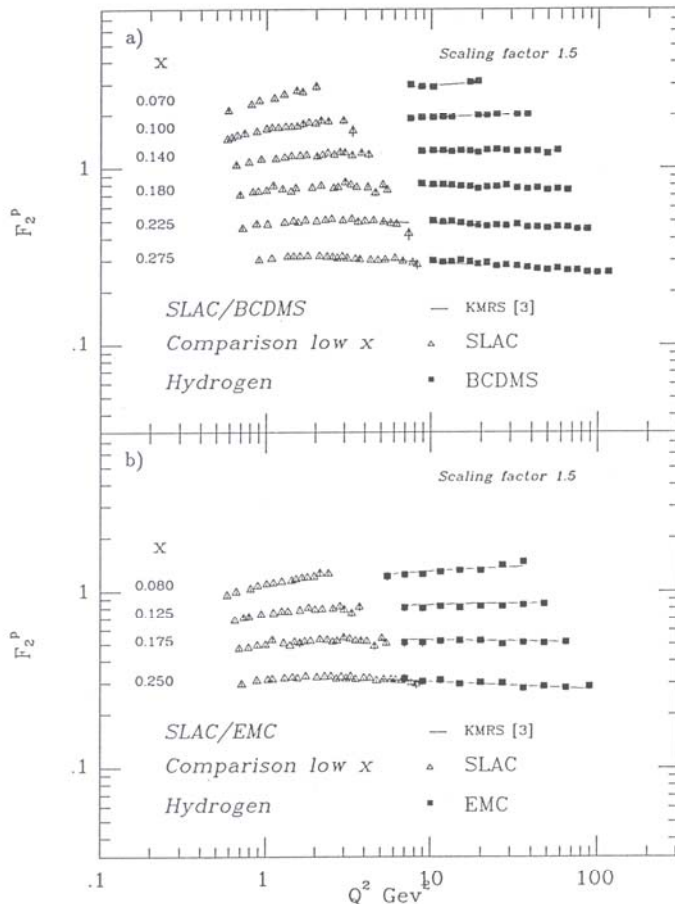


Figure 8. The SLAC re-analysed proton structure function data (open triangles) compared (a) with the BCDMS data, and (b) with the EMC data for the lower range. The SLAC data are binned appropriately for each comparison. The BCDMS data have been scaled down by 2 %, and the EMC up by 8 %. The individual x ranges are scaled by a factor of 1.5 to make them easily visible. This the scale is correct only for the highest x data on each plot.

Proton structure function
 $F_2(x, Q^2)$.

Open triangles: **SLAC** data

Solid squares:

a) BCDMS data

x values (from below):

0.275, 0.225, 0.180, 0.140,
 0.100, 0.070

b) EMC data

x values (from below):

0.250, 0.175, 0.125, 0.080

the individual x ranges are
 scaled by a factor of 1.5 to
 separate them from each other.

Note the slight Q^2 dependence:
 scaling violation!

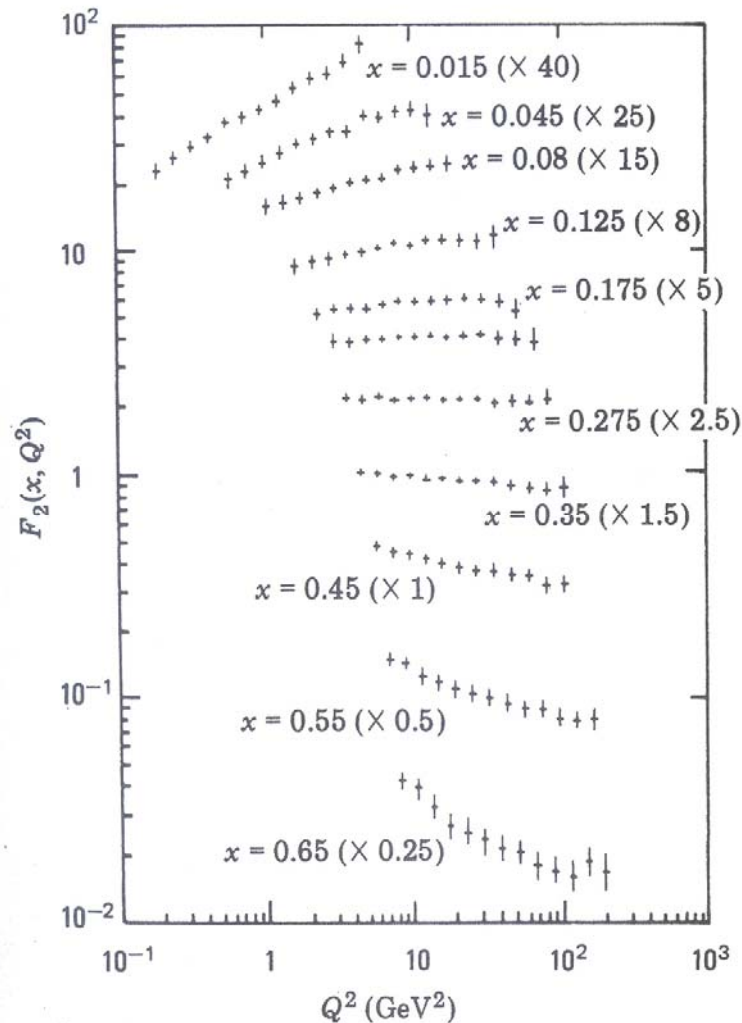


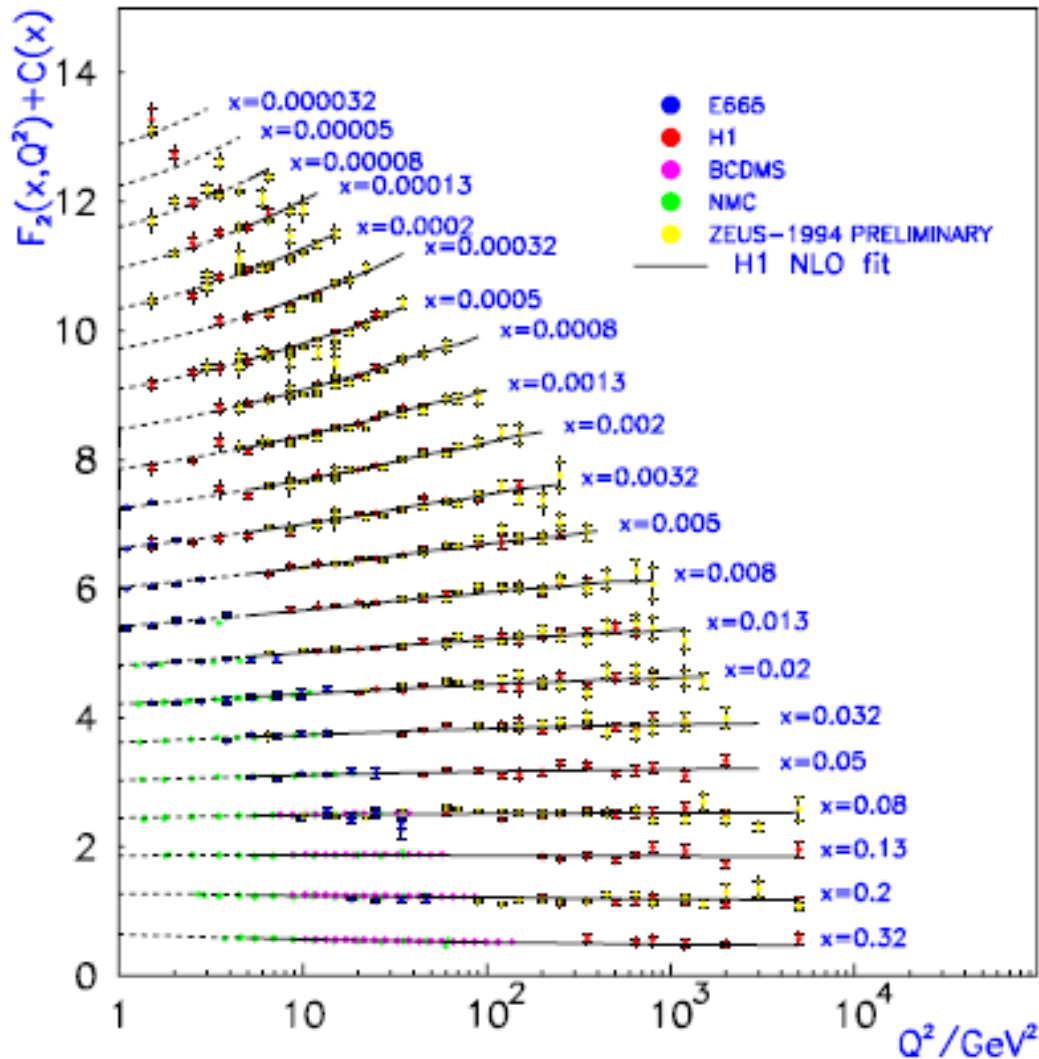
Fig. 5.6 The structure function $F_2^{\nu N}$ from the CDHSW collaboration (Berge *et al.* 1989).

Nucleon structure function
 $F_2^{\nu N}$ measured in
 neutrino-nucleon DIS

Data of CDHSW collaboration,
 1989

The individual x ranges are
 scaled by the factors shown
 to separate them from each
 other.

Clearly seen is the scaling violation
 at the lowest and highest x values



Compilation of data on the proton structure function $F_2^p(x, Q^2)$

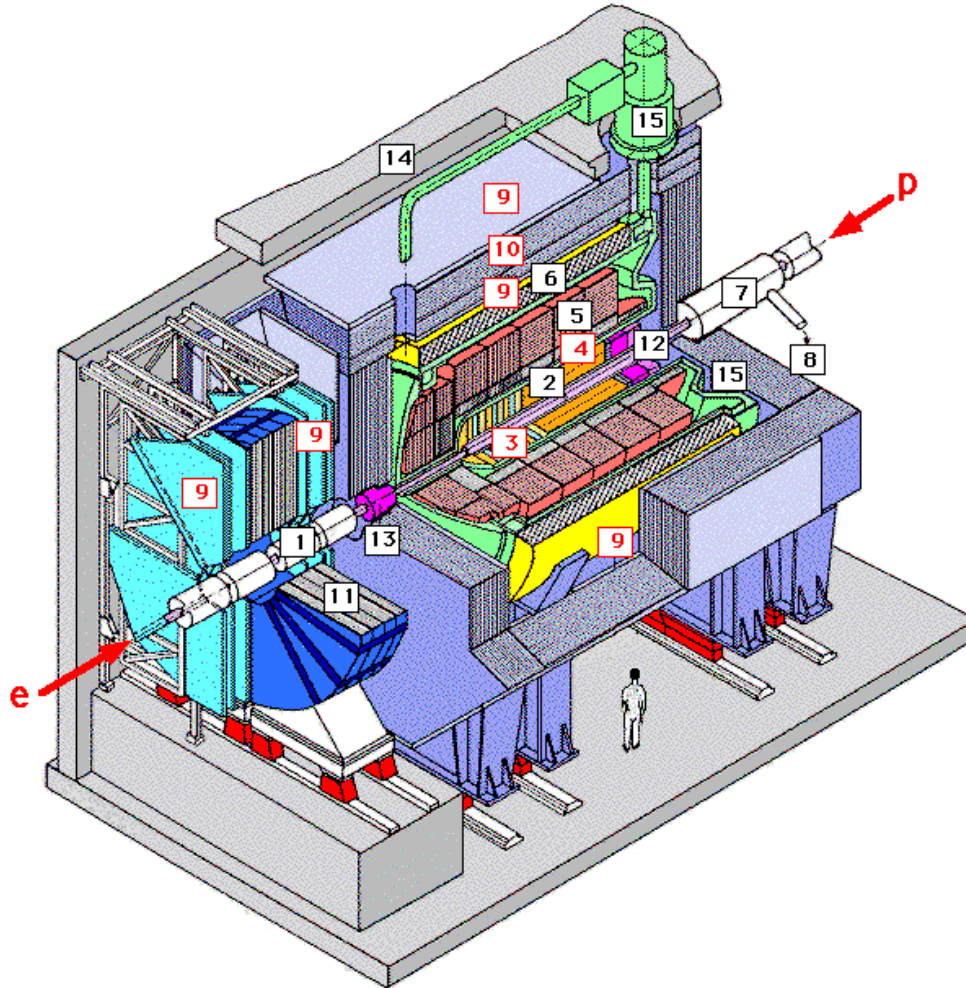
Experiments:

E666	(Fermilab)
H1	(DESY)
BCDMS	(CERN)
NMC	(CERN)
ZEUS	(DESY)

an amount $C(x)$ is added “by hand” to F_2 of each x range to separate the data sets.

Note the scaling violation at the lowest values of x

The H1 detector at the ep collider HERA (DESY):



Electrons of 30 GeV are colliding with protons of 820 GeV, giving

$$E_{\text{cm}} = 313 \text{ GeV}$$

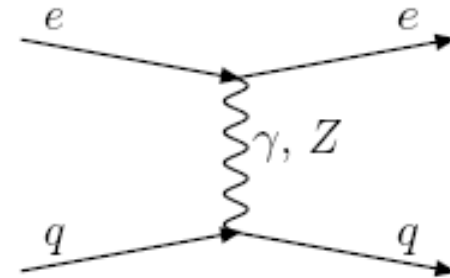
and

$$Q^2_{\text{max}} = 98\,400 \text{ GeV}^2$$

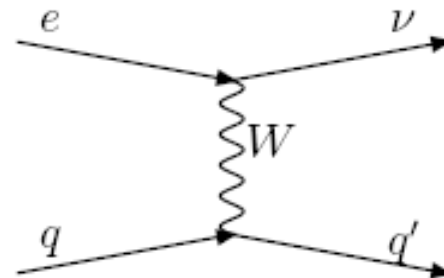
DIS by W and Z exchange are comparable to DIS by photon exchange at this energy.

The asymmetry of the colliding system leads to an asymmetric design of the detector.

In a **neutral current event** the transverse momentum is balanced, indicating that the hard electron-quark collision proceeds by photon or Z boson exchange:



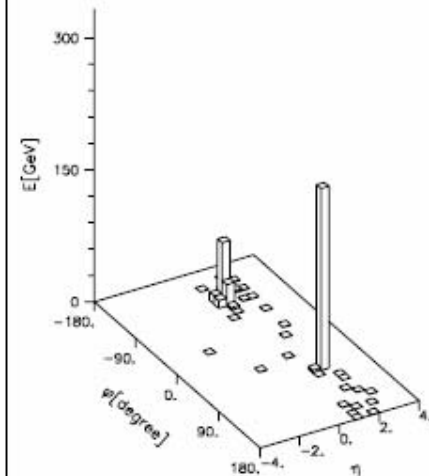
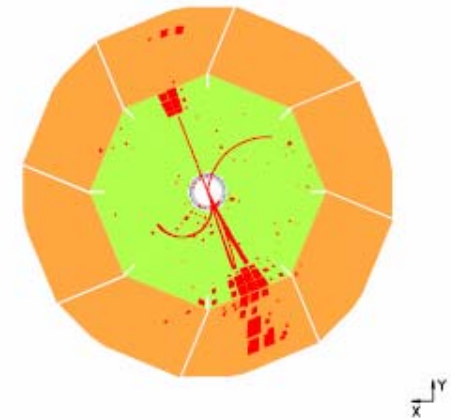
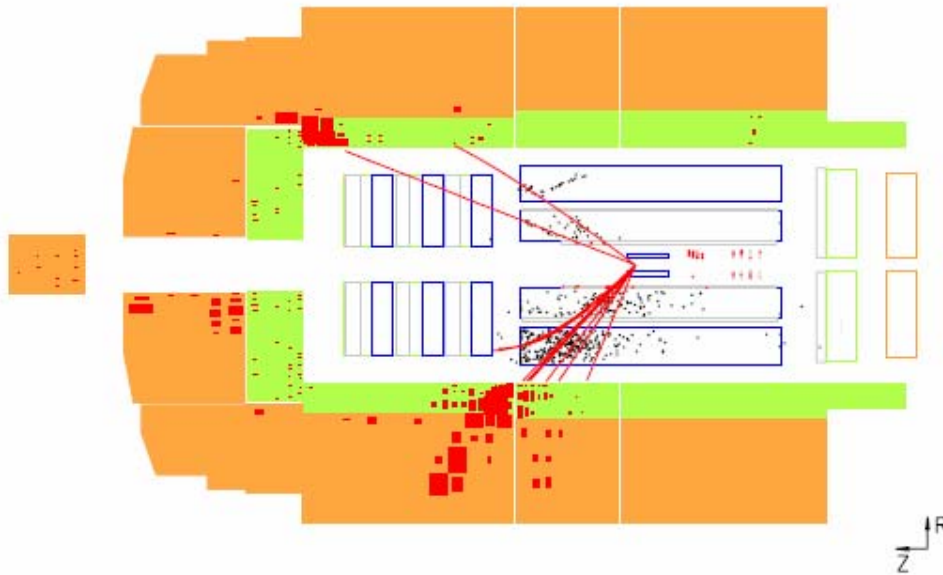
In a **charged current event**, the electron-quark collision proceeds by W boson exchange; the outgoing lepton is a neutrino and is not detected, therefore **the measured transverse momentum is not balanced**:



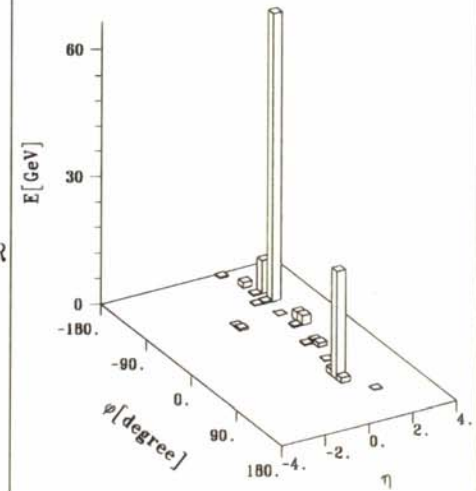
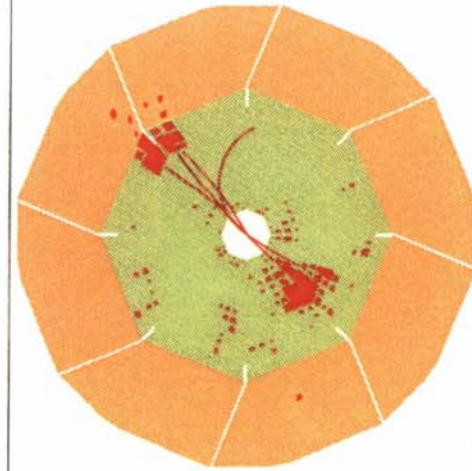
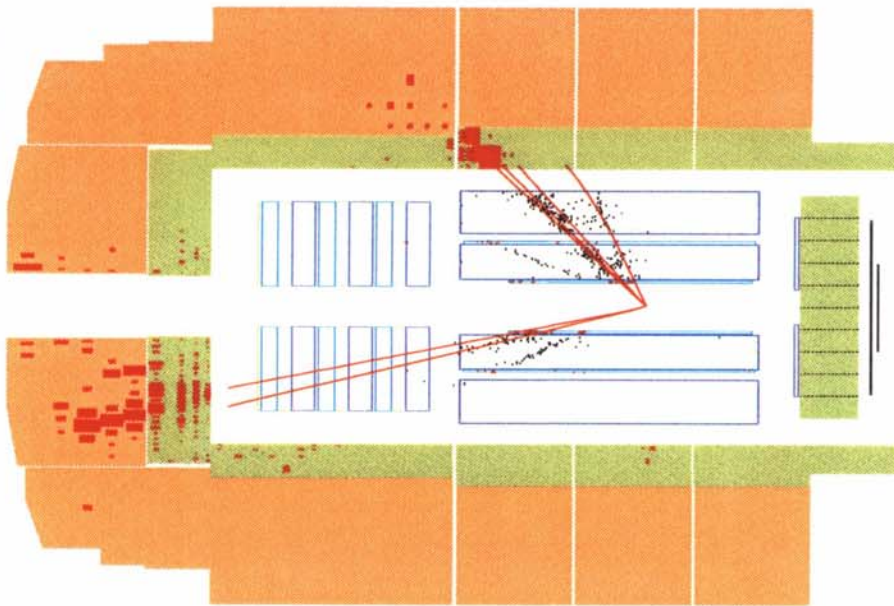
The following pictures are computer reconstructions of DIS events as seen by the H1 detector at DESY.



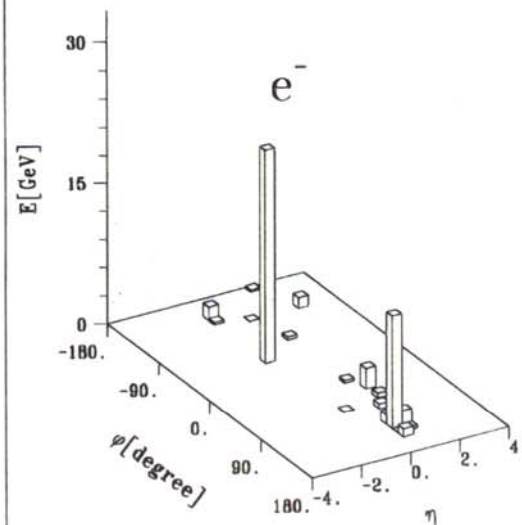
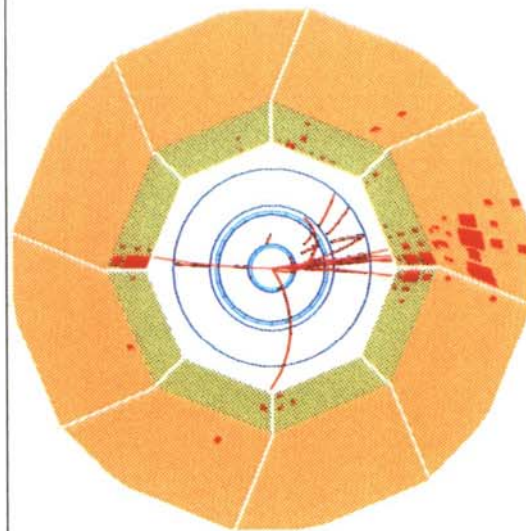
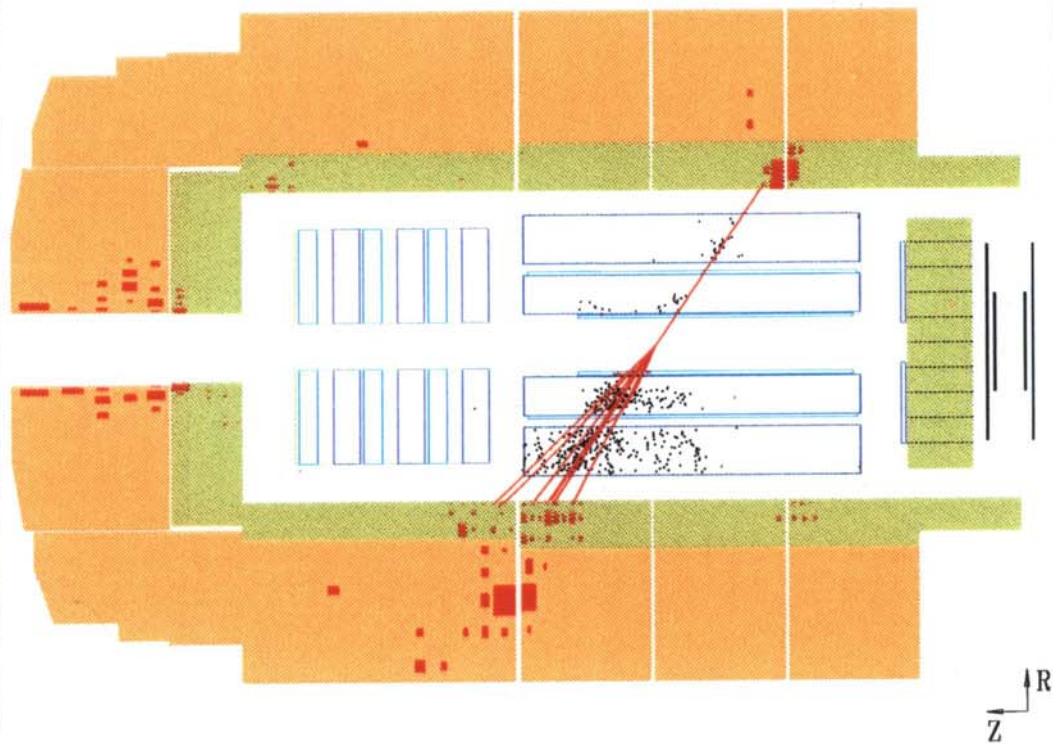
$Q^{*2} = 22068 \text{ GeV}^{*2}$, $y = 0.74$



Neutral current event in the H1 detector

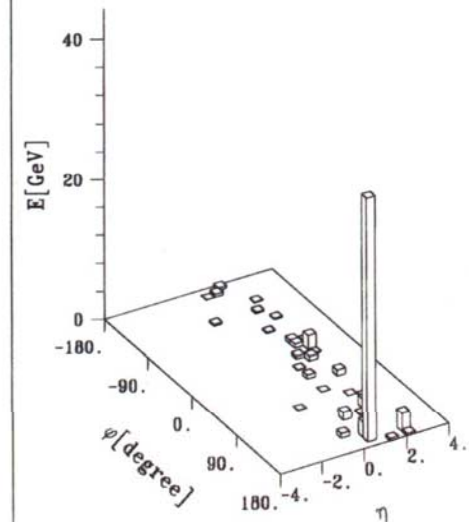
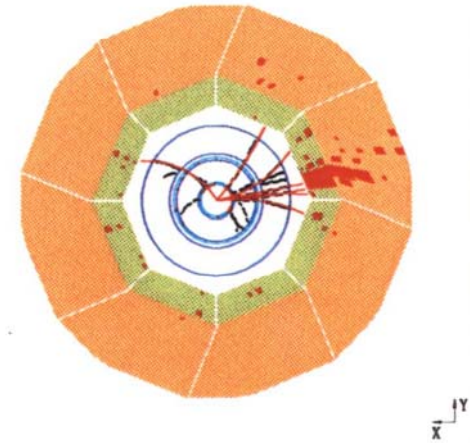


Neutral current event in the H1 detector



CC Event

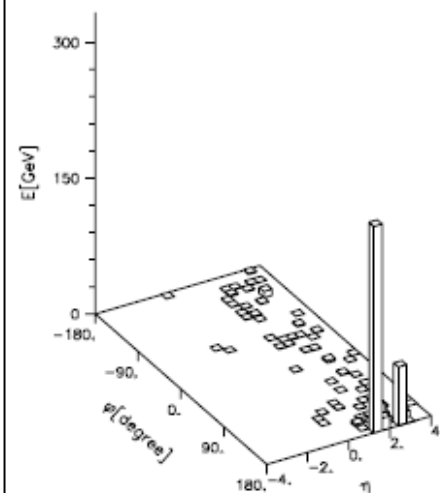
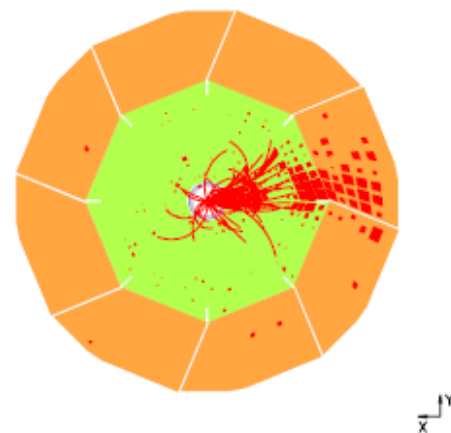
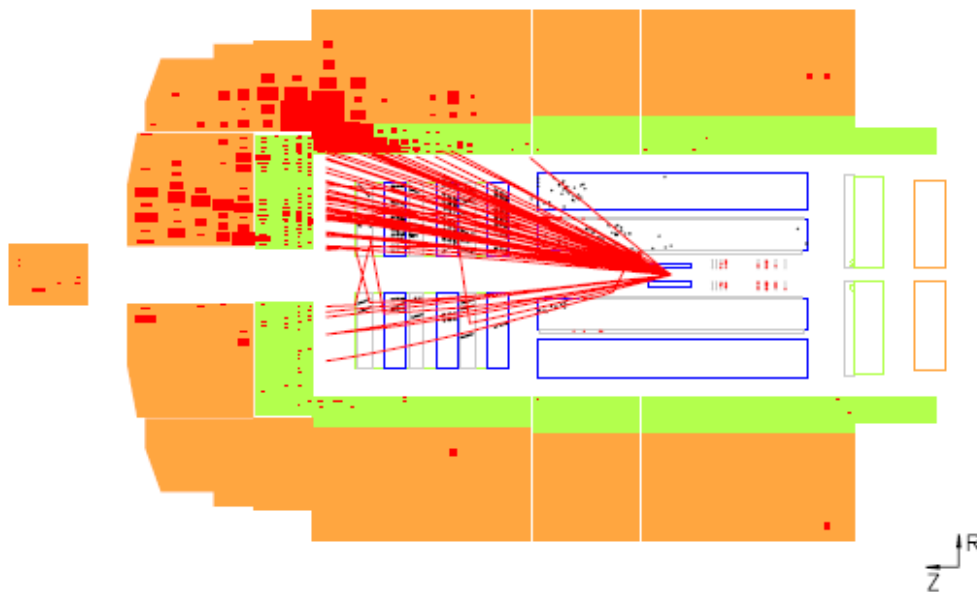
$$P_{t \text{ miss}} = 32 \text{ GeV} \quad Q^2 = 1800 \text{ GeV}^2$$



Charged current event: note the imbalance of transverse momentum.



Pt=139 Q2=41067 x=0.77 y=0.53



Charged current event in the H1 detector

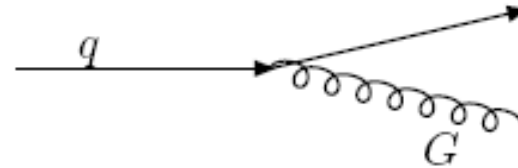
Scaling violation; gluons

Exact scaling holds if the quarks are free. In reality they are not free: quarks are held together in hadrons by *gluons*.

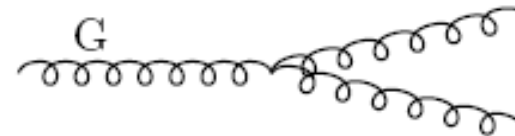
Gluons are the carriers of the strong interaction; they couple to the quarks and to each other. This gives rise to the following elementary processes:

Elementary Parton Processes:

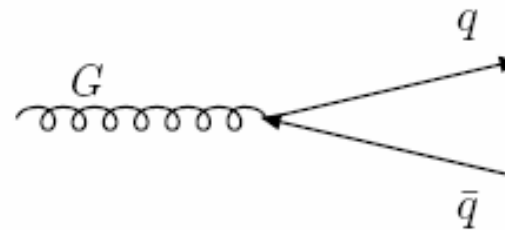
a) Quark emitting gluon:



b) Gluon splitting into 2 gluons:

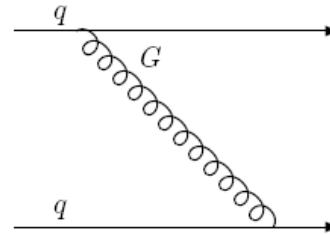


c) Gluon splitting into quark-antiquark pair:

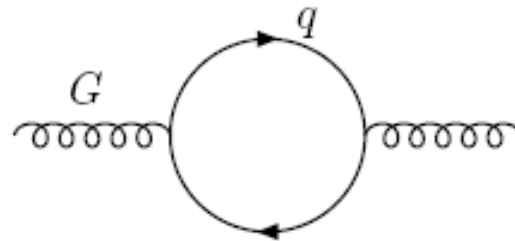


Build-up of gluon – quark – antiquark sea from elementary processes:

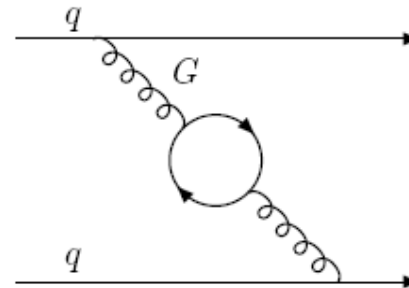
(ii) Gluon exchange between quark lines



(ii) Gluon splitting into quark-antiquark pair followed by recombination:



(iii) Gluon exchange with gluon splitting into quark-antiquark pair followed by recombination:



and these combine into more and more complicated processes

Revised Parton Picture of the Proton

proton = valence quarks (uud) + *gluon-quark-antiquark sea*

The proton charge is the sum of the valence quark charges:

the charge of the u quark is $+2/3e$

the charge of the d quark is $-1/3e$

the **parton sea** is neutral

A theoretical framework to describe the proton structure
(and the structure of all hadrons) is

Quantum Chromodynamics (QCD)

But there are great computational difficulties ...

If the structure function is known at some value of Q^2 , then one can reasonably accurately calculate the SF at all Q^2 using the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equations.

(this is what is shown in the figure of compilation of structure function data)

Cross sections for DIS processes, $i = \text{NC or CC}$:

$$\frac{d^2\sigma^i}{dx dy} = \frac{4\pi\alpha^2}{xyQ^2} \eta^i \left\{ \left(1 - y - \frac{x^2 y^2 M^2}{Q^2} \right) F_2^i + y^2 x F_1^i \mp \left(y - \frac{y^2}{2} \right) x F_3^i \right\},$$

$$\eta^{\text{NC}} = 1 \quad \eta_W = \frac{1}{2} \left(\frac{G_F M_W^2}{4\pi\alpha} \frac{Q^2}{Q^2 + M_W^2} \right)^2$$

NC structure functions:

$$[F_2^\gamma, F_2^{\gamma Z}, F_2^Z] = x \sum_q [e_q^2, 2e_q g_V^q, g_V^{q2} + g_A^{q2}] (q + \bar{q}),$$

$$[F_3^\gamma, F_3^{\gamma Z}, F_3^Z] = \sum_a [0, 2e_q g_A^q, 2g_V^q g_A^q] (q - \bar{q}),$$

where $g_V^q = \pm \frac{1}{2} - 2e_q \sin^2 \theta_W$ and $g_A^q = \pm \frac{1}{2}$, $+(-)$ for **u (d)** –type quarks

θ_W is the weak mixing angle: $e = g \sin \theta_W$

$$\sin^2 \theta_W = 1 - M_W^2 / M_Z^2 \cong 0.23$$

$$q = u, d, s, \dots$$

$$G_F = 1.166 \times 10^{-5} \text{ GeV}^2 \quad \textit{Fermi constant}$$

In the “naïve” quark-parton model, the parton densities are functions of the Bjorken-Feynman scaling variable x only. The observed scaling violation means that they are functions of x and Q^2 . The Q^2 dependence cannot be calculated, but if the x dependence is taken from experiment at some value of Q^2 , then the x dependence at other Q^2 values can be calculated using the QCD evolution equations of Dokshitzer, Gribov, Lipatov, Altarelli and Parisi (DGLAP equations).

DGLAP QCD evolution equation for parton densities:

$$\frac{\partial f}{\partial \ln \mu^2} \sim \frac{\alpha_s(\mu^2)}{2\pi} (P \otimes f) ,$$

$$P \otimes f = \int_x^1 \frac{dy}{y} P(y) f\left(\frac{x}{y}\right)$$

$P(y)$ are splitting functions which can be calculated within the framework of perturbative QCD (pQCD).

In lowest order pQCD the splitting functions are:

$$P_{qq} = \frac{4}{3} \left[\frac{1+x^2}{(1-x)} \right]_+ = \frac{4}{3} \left[\frac{1+x^2}{(1-x)_+} \right] + 2\delta(1-x) ,$$

$$P_{qg} = \frac{1}{2} \left[x^2 + (1-x)^2 \right] ,$$

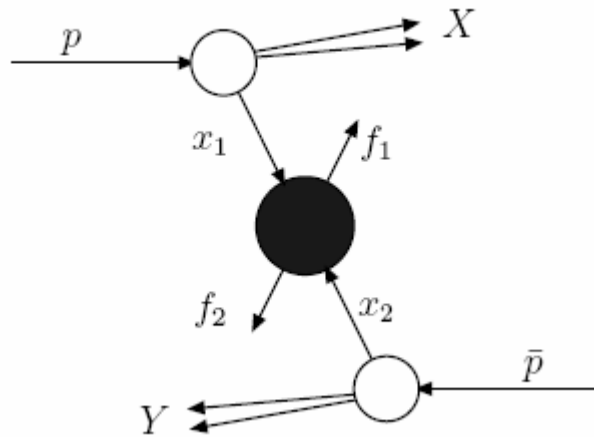
$$P_{gq} = \frac{4}{3} \left[\frac{1+(1-x)^2}{x} \right] ,$$

$$P_{gg} = 6 \left[\frac{1-x}{x} + x(1-x) + \frac{x}{(1-x)_+} \right] \\ + \left[\frac{11}{2} - \frac{n_f}{3} \right] \delta(1-x) ,$$

where

$$\int_0^1 dx f(x) [F(x)]_+ = \int_0^1 dx (f(x) - f(1))F(x)$$

Diagram of deep inelastic p-anti p collision:



a parton from the proton, that carries a fractional momentum x_1 , makes a hard collision with a parton from the anti proton, that carries a fractional momentum x_2 . The final state of this collision are fermions f_1 and f_2 .

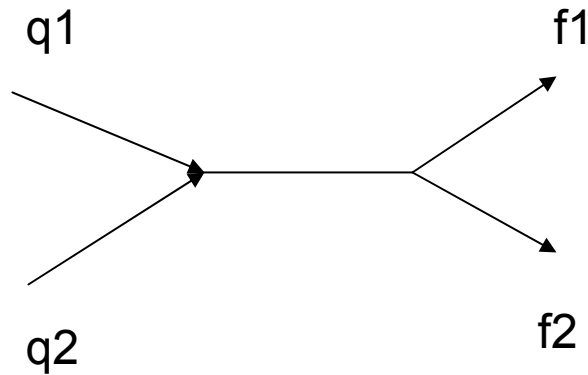
The remaining partonic system of the proton and anti proton hadronise into systems X and Y

A particular case of the hard collision is the reaction

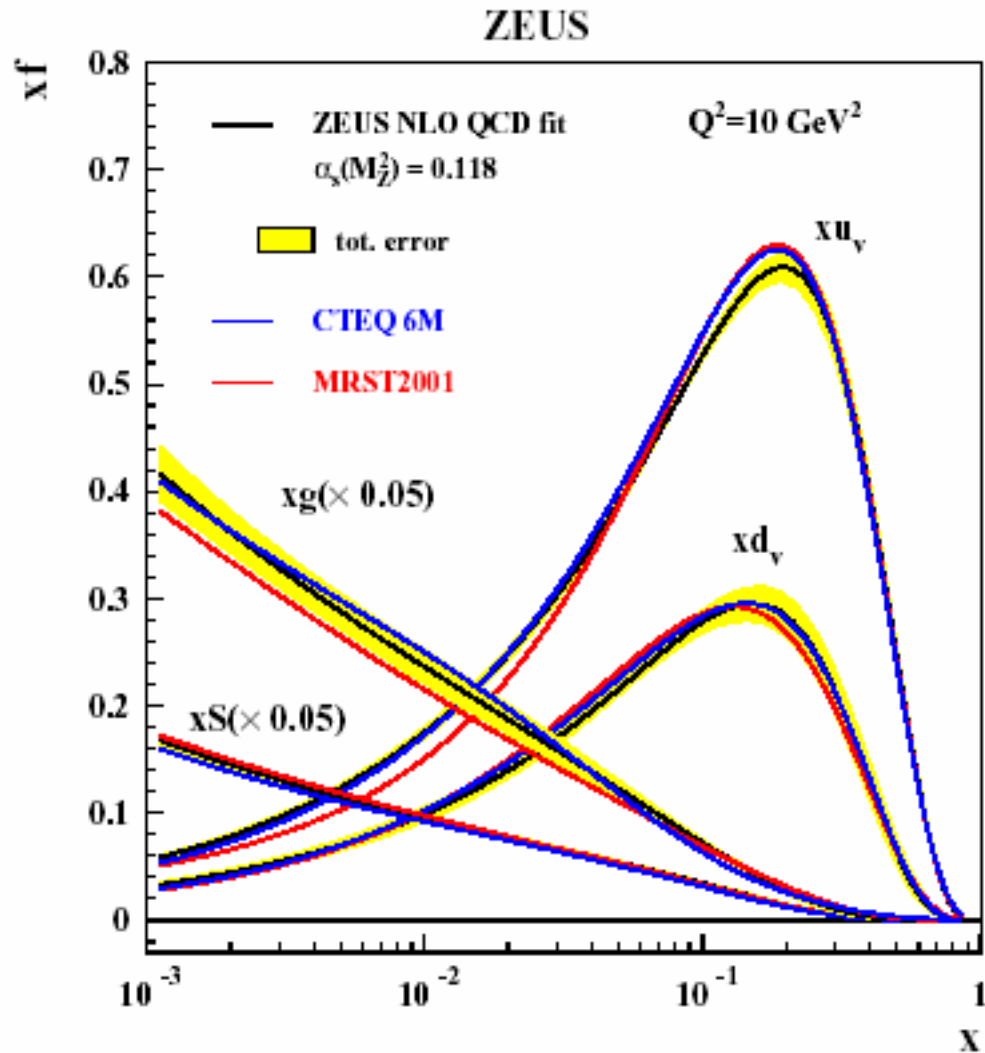
$$q\bar{q} \rightarrow Z \rightarrow e^+e^-$$

first seen in the UA1 detector at CERN in 1982. Such collisions are also seen at the Tevatron (Fermilab) and will be seen at the LHC (CERN).

A process of the type



where q_1 and q_2 are quarks from different hadrons, and f_1 and f_2 are final state fermions, are called Drell-Yan processes. They are particularly important in proton-proton and proton-antiproton collisions.



The data from all DIS experiments taken together in “Global Fits” using a QCD based theoretical framework yield distributions of the individual partons.

Shown here are the momentum distributions of the valence u and d quarks, quark-antiquark sea and gluons.

Note the scaling of the gluon and sea distributions!

The fits of three different groups are in good agreement