# High Energy Physics

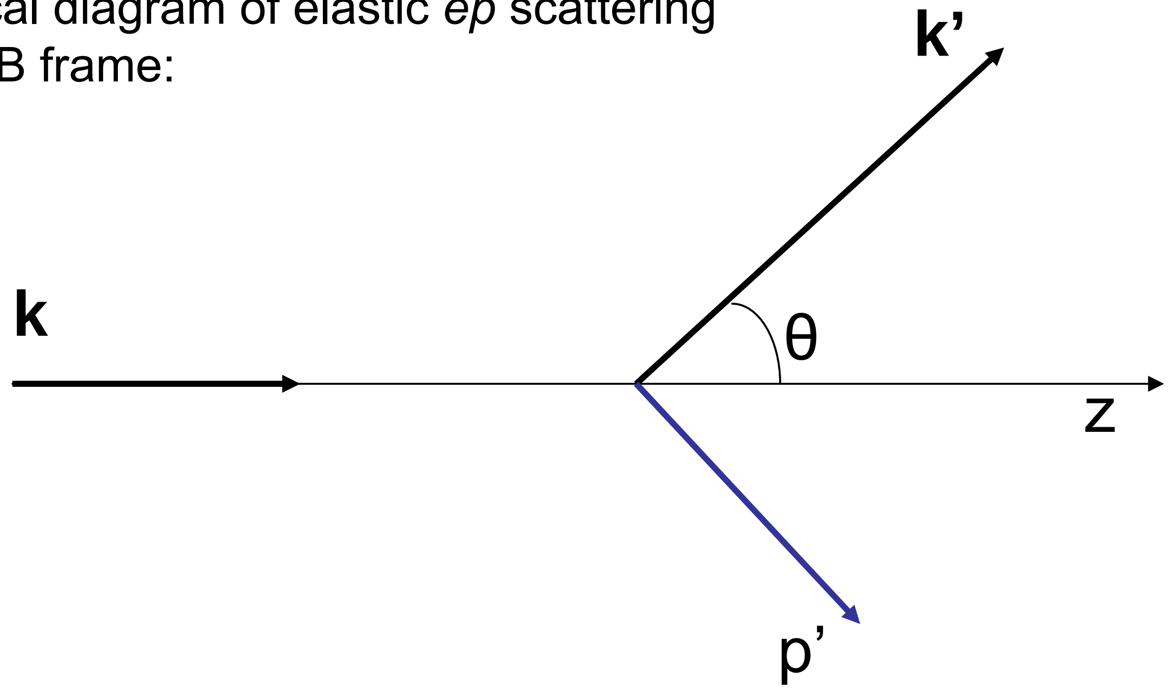
## Lecture 8

Part 1: Elastic electron-proton scattering

Part 2: Deep Inelastic Scattering

# Part 1: Elastic electron-proton Scattering

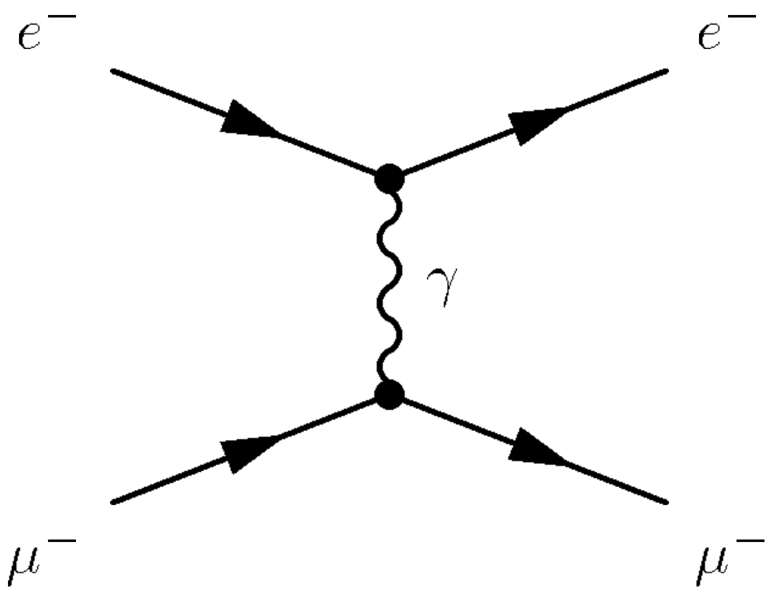
Kinematical diagram of elastic *ep* scattering in the LAB frame:



# Feynman diagram of elastic electron-muon scattering

Electrons and muons are leptons; the electromagnetic interaction is by photon exchange

*The lepton-photon coupling is point-like*




***Kinematics* (units: *c*=1)*:***

Electron:

initial energy and momentum:

$$\omega, \vec{k}$$

final energy and momentum:

$$\omega', \vec{k}'$$

muon:

$$E, \vec{p}; \quad E', \vec{p}'$$

$E^2 = \vec{p}^{\,2} + m^2$ and similarly for *E'*, *ω* and *ω'*

***4-vector:***

$$\mathrm{p} = ( E \ , \ \vec{p} \ )$$

$$\mathrm{p}^2 = E^2 - \vec{p}^{\,2} = m_\mu^2 = \text{invariant}$$

$$\mathrm{k}^2 = \omega^2 - \vec{k}^{\,2} = m_e^2$$

***Energy and momentum conservation:***

$$E + \omega = E' + \omega'; \quad \vec{p} + \vec{k} = \vec{p}\,' + \vec{k}\,'$$

Combine into 4-momentum conservation:

$$\mathrm{p} + \mathrm{k} = \mathrm{p}' + \mathrm{k}'$$

Photon 4-momentum: $\mathrm{q} = \mathrm{k} - \mathrm{k}' = \mathrm{p}' - \mathrm{p}$

## Invariant square of the photon 4-momentum:

$$q^2 = (k - k')^2 = k^2 + k'^2 - 2k \cdot k'$$
$$= 2m_e^2 - 2\left(\omega\omega' - \vec{k} \cdot \vec{k}'\right)$$

But the electron mass is negligibly small, hence

$$\omega = \left|\vec{k}\right| \equiv k,\ \omega' = \left|\vec{k}'\right| \equiv k'$$
$$q^2 = -2kk'(1 - \cos\theta) < 0 \quad \text{"}space - like\text{"}$$

The mass of a real photon is zero;
a photon with a non-zero mass is called *virtual photon.*

The electromagnetic interaction takes place by the exchange of a virtual photon.

Some of the following material is from R.E. Taylor's Nobel Lecture:

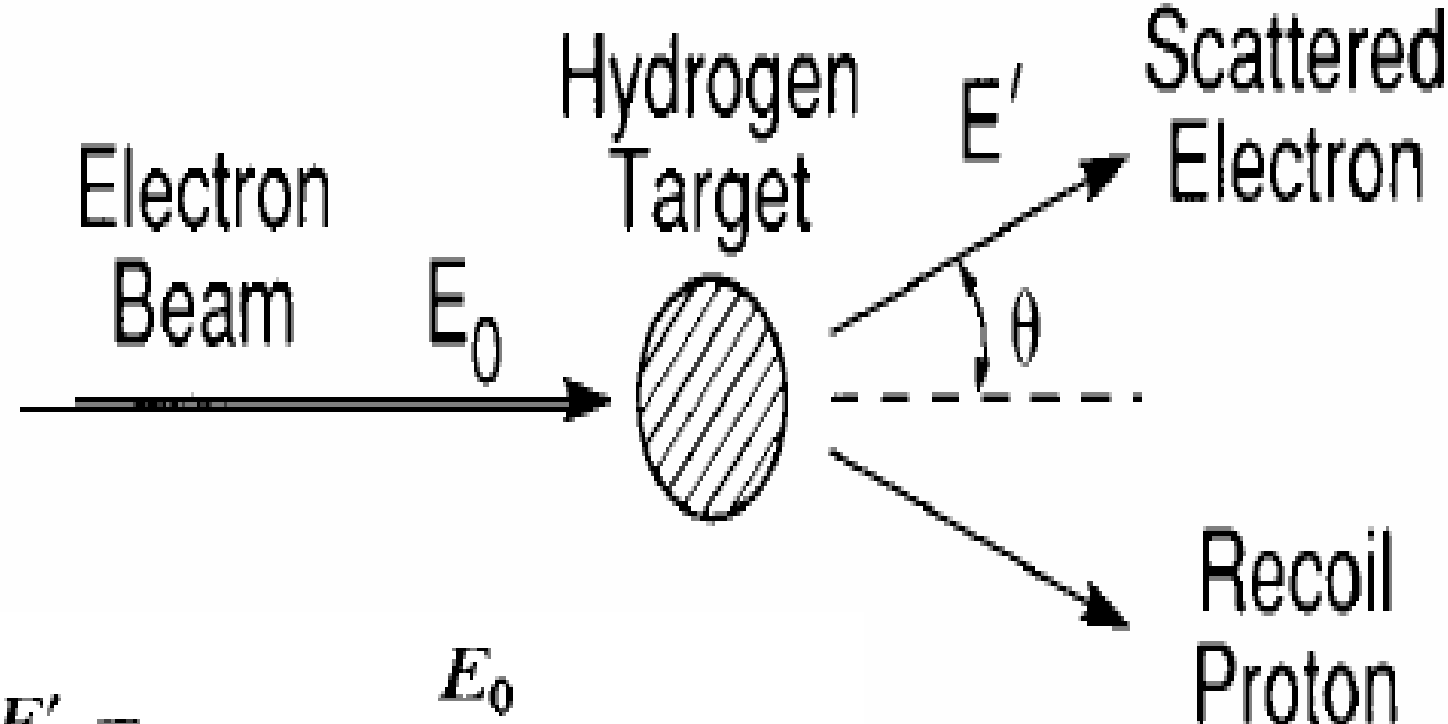
# DEEP INELASTIC SCATTERING: THE EARLY YEARS

Nobel Lecture, December 8, 1990

RICHARD E. TAYLOR

Stanford Linear Accelerator Center, Stanford, California, USA

## Electron-proton elastic scattering:




$$E' = \frac{E_0}{1 + \frac{2E_0}{M}\sin^2\theta/2}$$

## Derivation:

$$\mathrm{P}=\left(M,\vec{0}\right),\ \mathrm{P}'=\left(P'^{0},\vec{P}'\right),\ \mathrm{p}=\left(E,\vec{p}\right),\ \mathrm{p}'=\left(E',\vec{p}'\right);$$

4 – momentum conservation:

$$\mathrm{P+p=P'+p'}$$

$$\mathrm{P}'^2=M^2=\left(\mathrm{P+p-p'}\right)^2=M^2+\left(\mathrm{p-p'}\right)^2+2M\left(E-E'\right)$$

but

$$\left(\mathrm{p-p'}\right)^2=2m_e^2-2\mathrm{p}\cdot\mathrm{p}'=-2EE'\left(1-\cos\theta\right)$$

where in the last step we have neglected the electron mass,

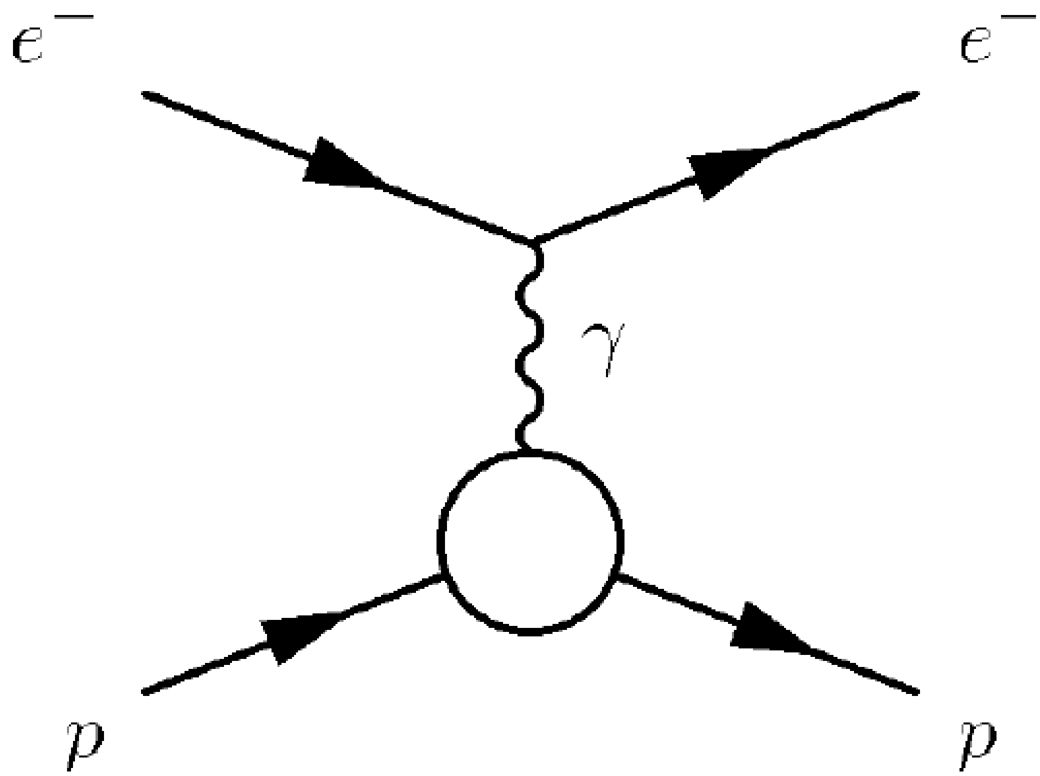
hence

$$EE'\left(1-\cos\theta\right)=M\left(E-E'\right)$$

and finally

$$E'=\frac{E}{1+\left(2E/M\right)\sin^2\left(\theta/2\right)}$$

# Feynman diagram of elastic ep scattering:




the proton is of finite size; this is taken into account by ***form factors:*** **one** ***electric*** **and one** ***magnetic ff***

Differential cross section of elastic electron-proton scattering (**Rosenbluth** formula): based on relativistic quantum mechanics

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_0^2 \sin^4 \theta/2} \cdot \cos^2 \theta/2 \cdot \frac{E'}{E_0} \left[ \frac{G_E^2 + \tau G_M^2}{1+\tau} + 2\tau G_M^2 \tan^2 \theta/2 \right]$$

The functions $G_E$ and $G_M$ take account of the size of the proton; they are called ***form factors***

$$G_E = G_E\left(Q^2\right), G_M = G_M\left(Q^2\right); \qquad G_E(0) = 1,\ G_M(0) = \mu_p$$

$Q^2$ is up to the sign the square of the 4-momentum transfer:

$$\boxed{Q^2 = 4E_0E' \sin^2 \theta/2} \qquad \boxed{\tau = Q^2 / 4m_p^2}$$

$\mu_p = 2.79$ is the proton ***anomalous magnetic moment***

From a simpler calculation for a proton without spin (and therefore without magnetic moment) and neglecting the recoil of the proton one finds

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_0^2 \sin^4 \frac{\theta}{2}} \cdot \cos^2 \frac{\theta}{2}$$

(Mott cross section)

Therefore we can write the Rosenbluth formula in the following form:

$$\boxed{\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \cdot \frac{E'}{E}\left[\frac{G_E^2 + \tau G_M^2}{1+\tau} + 2\tau G_M^2 \tan^2 \frac{\theta}{2}\right]}$$

The form factors cannot be calculated from first principles. Up to the present time they remain empirical functions, extracted from measured differential cross sections.

**Strategy:**

Measure $d\sigma/d\Omega$ at many angles $\theta$ and at many values of $Q^2$, hence extract form factors:

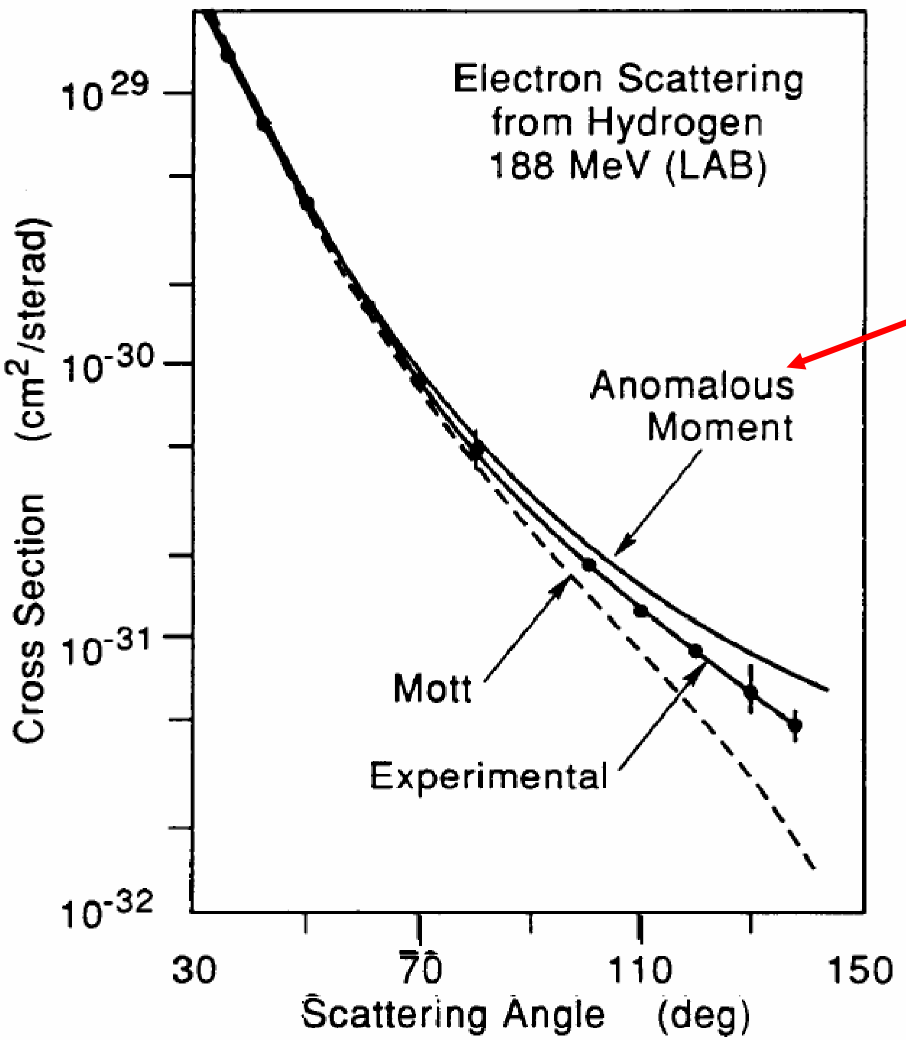



*Fig. 5.* **Elastic electron scattering cross sections from hydrogen compared with the Mott** scattering formula (electrons scattered from a particle with unit charge and no magnetic moment) and with the Rosenbluth cross section for a point proton with an anomalous magnetic moment. The data falls between the curves, showing that magnetic scattering is occurring but also indicating that the scattering is less than would be expected from a point proton.

Anomalous moment:
point-like proton with (anomalous) magnetic moment:

$$\boxed{\begin{aligned} G_E &= 1, \\ G_M &= \mu_p = 2.79 \end{aligned}}$$

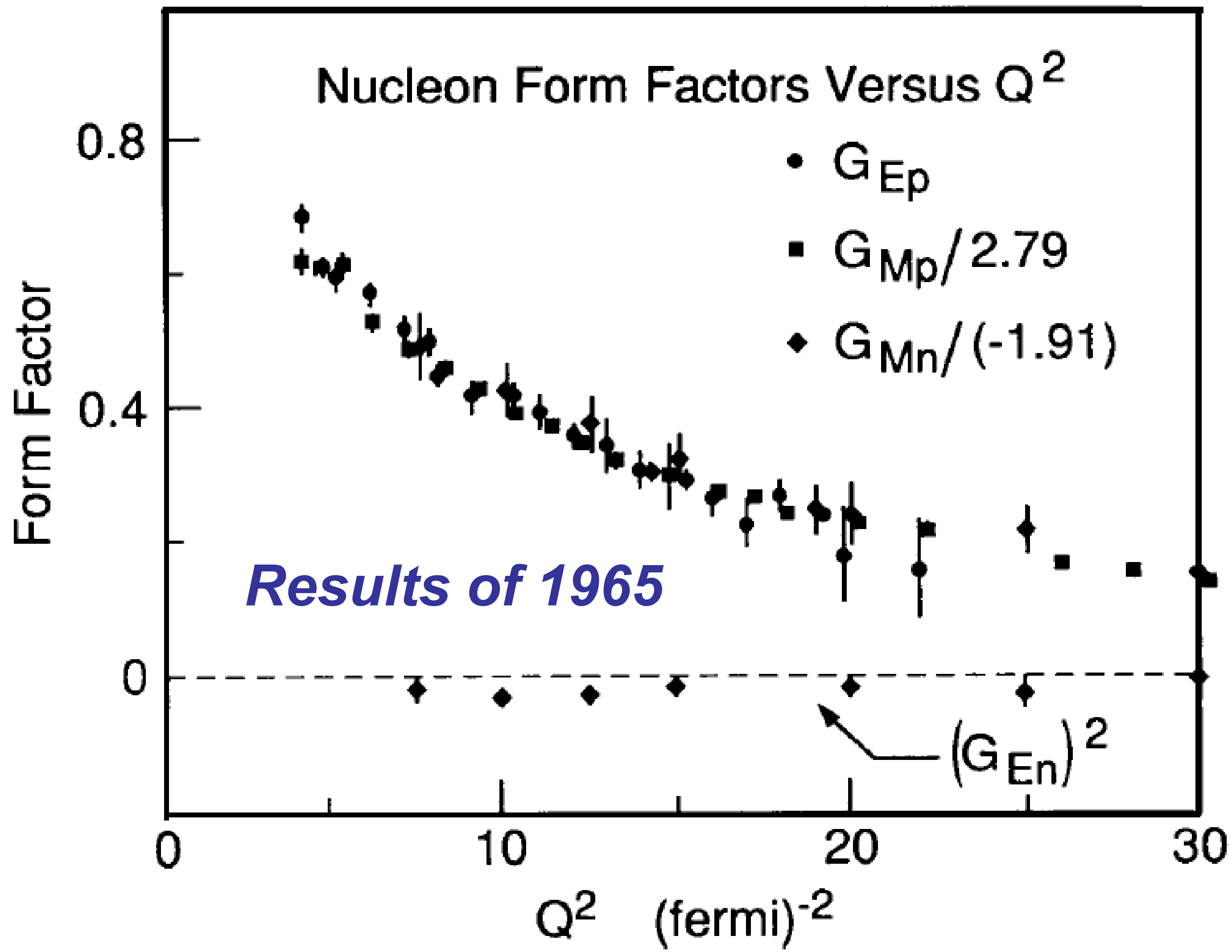



Fig. 23. Summary of results on nuclear form factors presented by the Stanford group at the 1965 "International Symposium on Electron and Photon Interactions at High Energies". (A momentum transfer of 1 $GeV^2$ is equivalent to 26 $Fermis^{-2}$.)

Quotation from Taylor:

The data continued to follow the so-called dipole model to a good approximation. By the Hamburg conference in 1965 there were no dissenters from the view that

$$G_{Ep} = \frac{G_{Mp}}{\mu_p} = \frac{G_{Mn}}{\mu_n},$$

$$G_{En} \cong 0 \text{ at large } Q^2,$$

and

$$G_{Ep}(Q^2) \cong \left( \frac{1}{1 + \frac{Q^2}{0.71 \text{ GeV}^2}} \right)^2 \text{ up to } Q^2 \sim 10 \text{ GeV}^2$$

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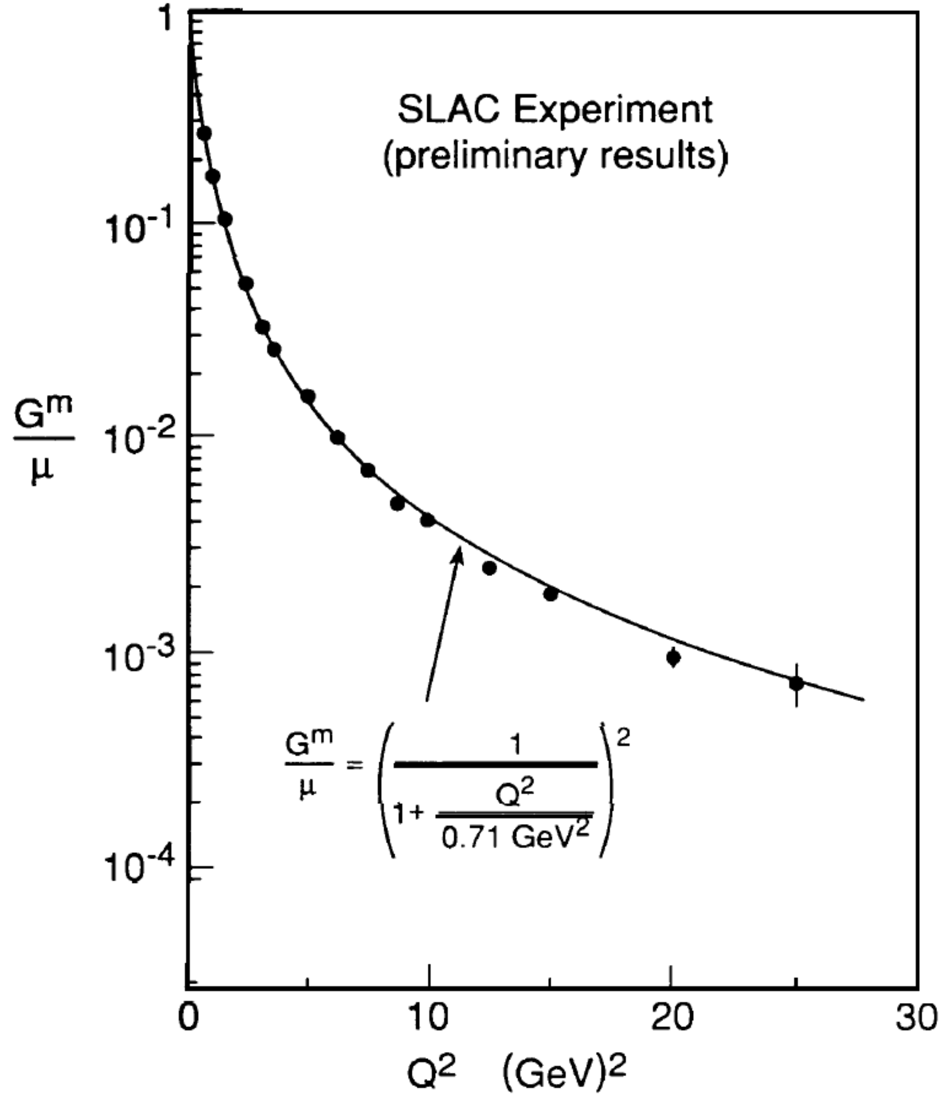



Fig. 36. Magnetic form factor measurement at SLAC in 1967. The dipole curve is the same as in Figure 25, here extended to $Q^2 = 25\ \mathrm{GeV}^2$. Again, the agreement is imperfect but the curve describes the general behavior of the data quite well.

Results of measuring the proton magnetic form factor (SLAC 1967)

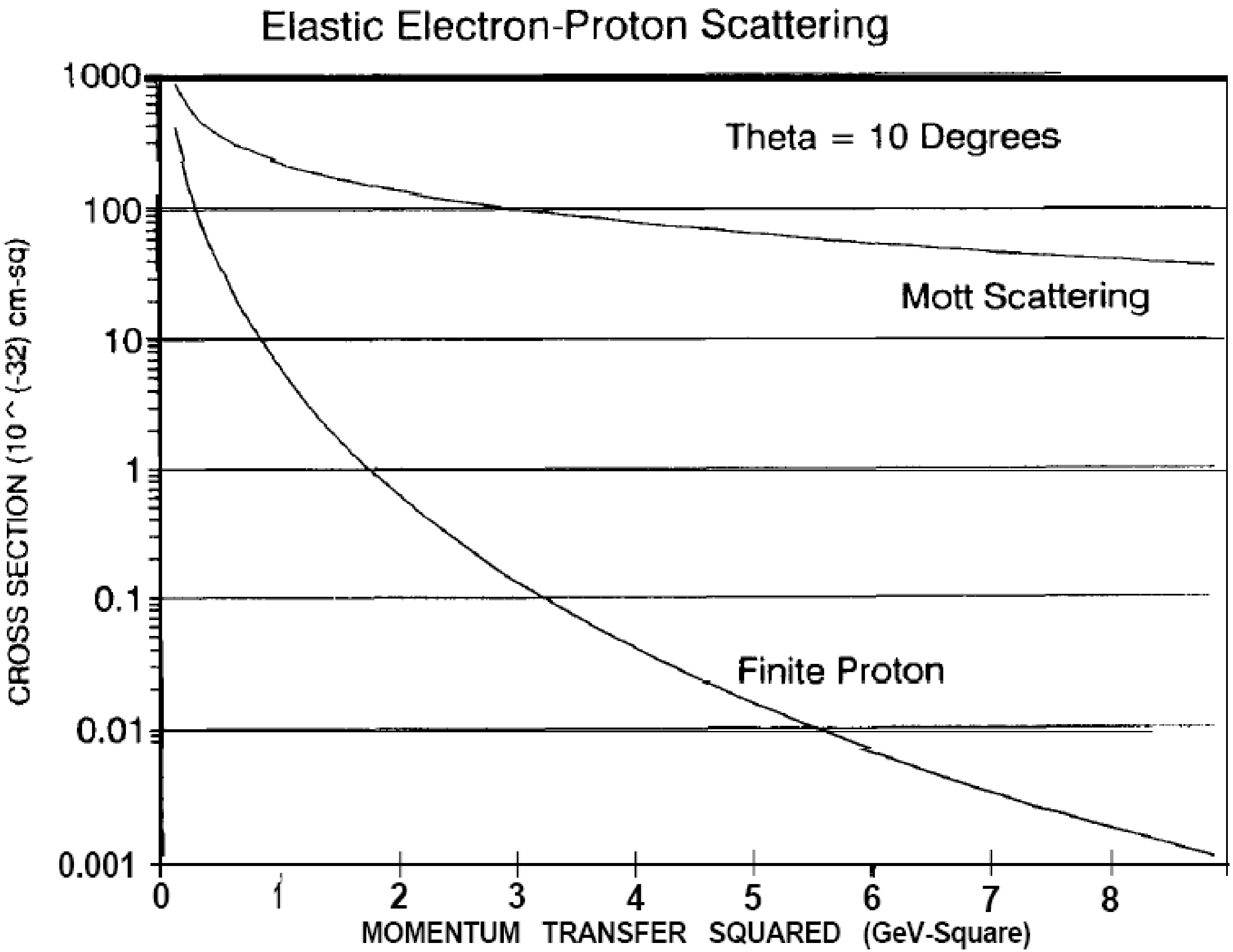



**Fig. 4. Elastic scattering cross sections for electrons from a "point" proton and for the actual proton. The differences are attributable to the finite sire of the proton.**

# Summary of Results on Elastic Electron-Proton Scattering

From basic principles of relativistic quantum mechanics, the differential cross section of electron-proton scattering, assuming the proton to be an extended object, is given by the Rosenbluth formula:

$$\boxed{\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \cdot \frac{E'}{E}\left[\frac{G_E^2 + \tau G_M^2}{1+\tau} + 2\tau G_M^2 \tan^2\frac{\theta}{2}\right]}$$

The form factors

$$\boxed{G_E = G_E\left(Q^2\right), G_M = G_M\left(Q^2\right);}$$

are empirical functions of the 4-momentum transfer. From experiment one finds

$$\boxed{G_E = \frac{G_M}{\mu_p} = \left(1 + \frac{Q^2}{a^2}\right)^{-2}, \quad a^2 = 0.71\,\text{GeV}^2}$$

(*dipole formula*)

These results formed the prejudice that the proton was a soft (“mushy”) extended object, possibly with a hard core surrounded by a cloud of mesons, mainly pions.

The SLAC-MIT team saw its objective in searching for the hard core of the proton.

This could be done exploiting the higher energy of the electron beam that became available with the commissioning of the 2-mile Linac: $E$ = 20 GeV

Linac is the 2 mile Stanford Linear Accelerator
Electrons are accelerated up to 20 GeV

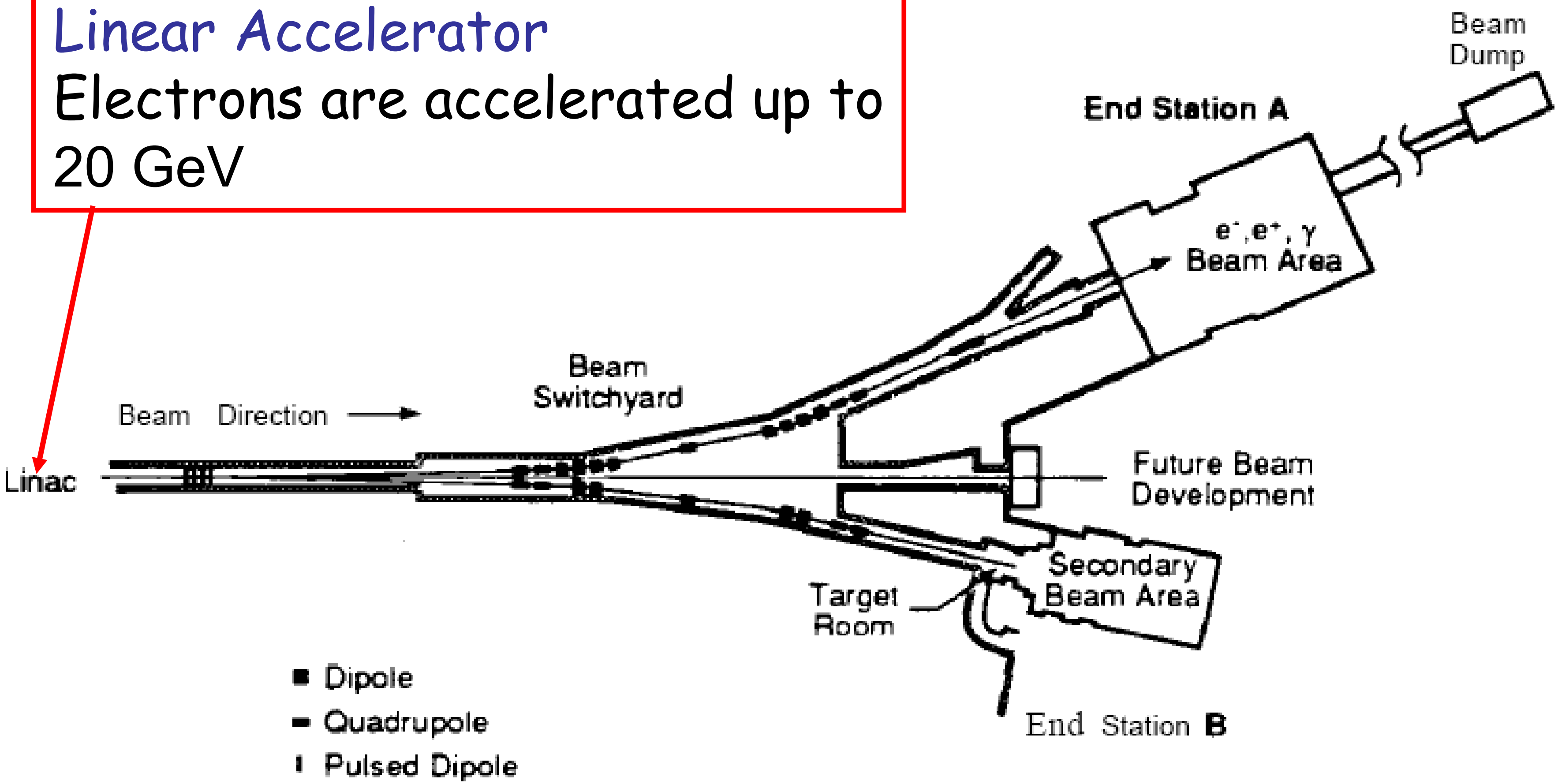
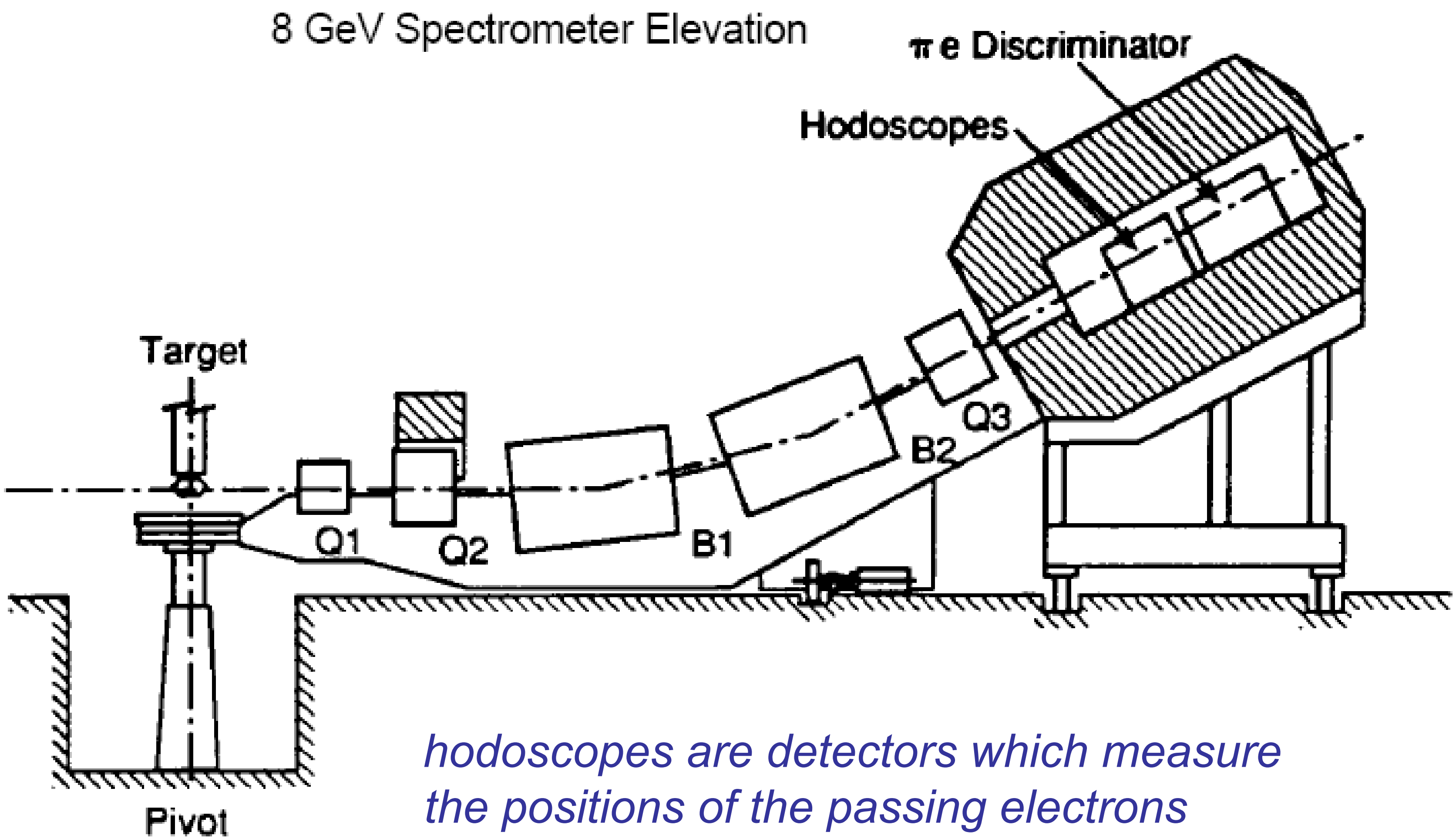



Fig. 12. Layout of the SLAC experimental areas and the beam switchyard.

Energies and momenta of the scattered electrons are measured in spectrometers:




*hodoscopes are detectors which measure the positions of the passing electrons*

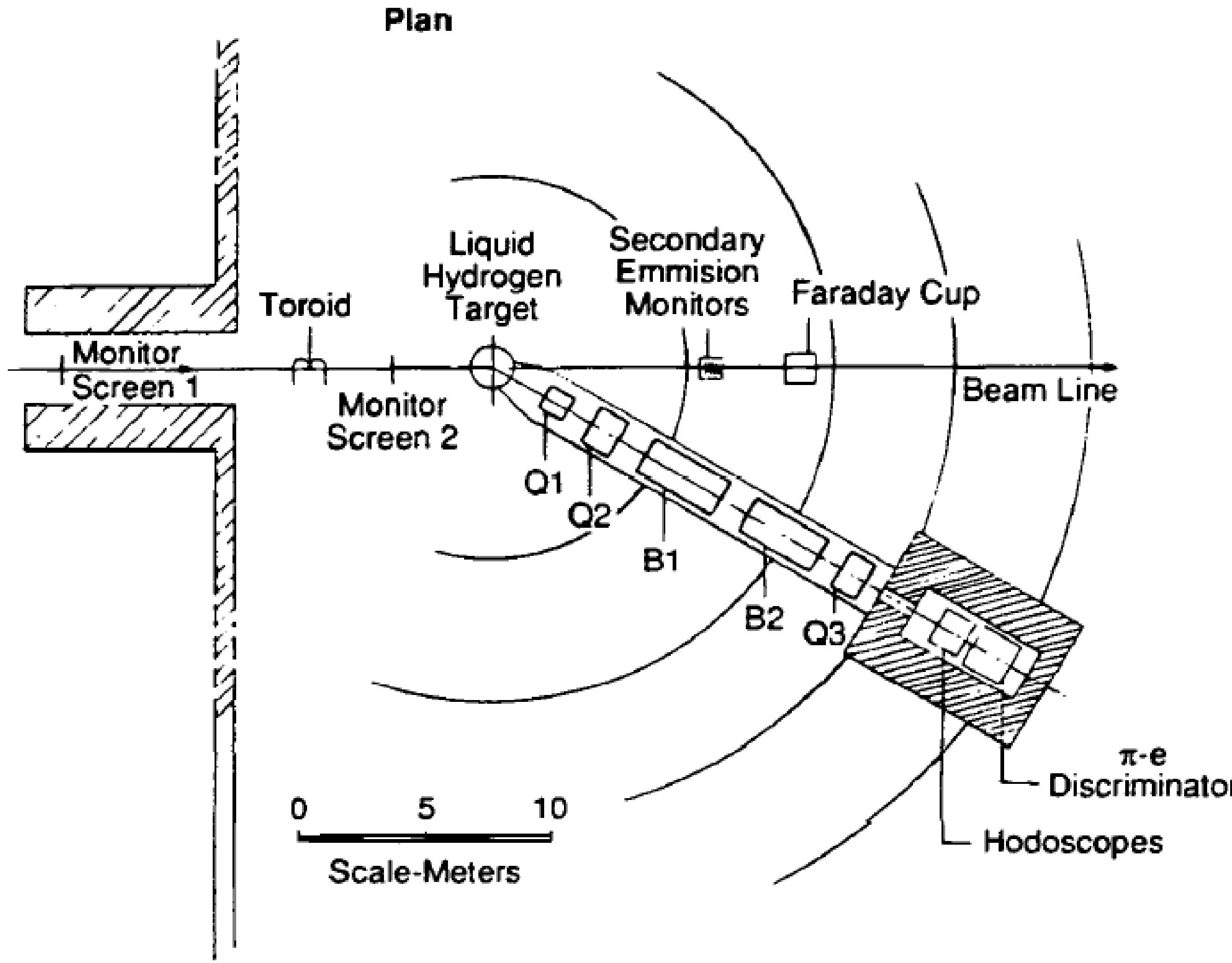



Fig. 13. Schematic drawings of the 8 GeV spectrometer. Five magnets (two bending magnets, (B), and three quadrupoles, (Q)) direct scattered particles into the detectors which are mounted in a heavily shielded enclosure. The whole assembly rides on the rails and can be pivoted about the target to change the angle of scattering of the detected electrons.

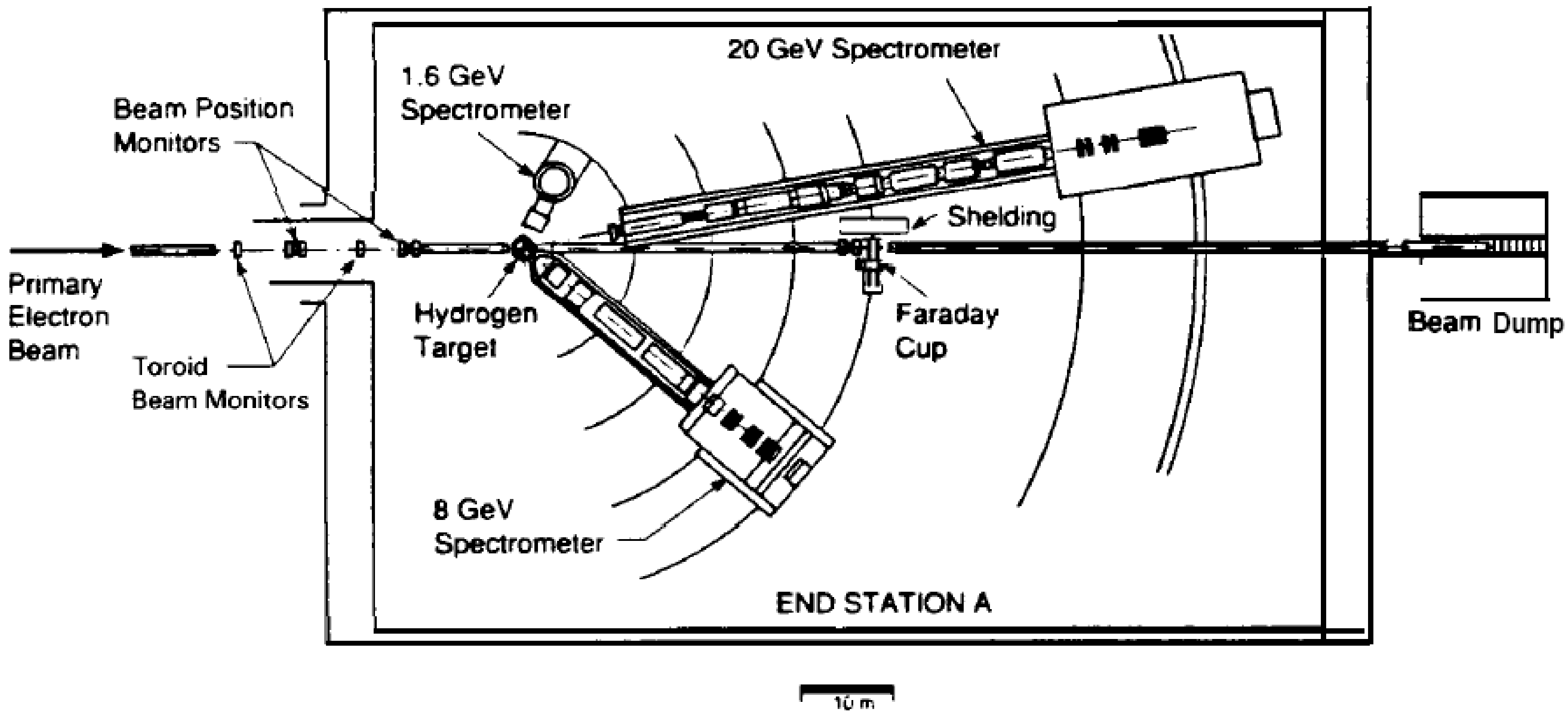



Fig. 14. Layout of spectrometers in End Station A. All three spectrometers can be rotated about the pivot. The 20 GeV spectrometer can be operated from about $1\frac{1}{2}°$ to 25°, the 8 GeV from about 12° to over 90°. The 1.6 GeV spectrometer coverage is from $\sim 50° - 150°$.

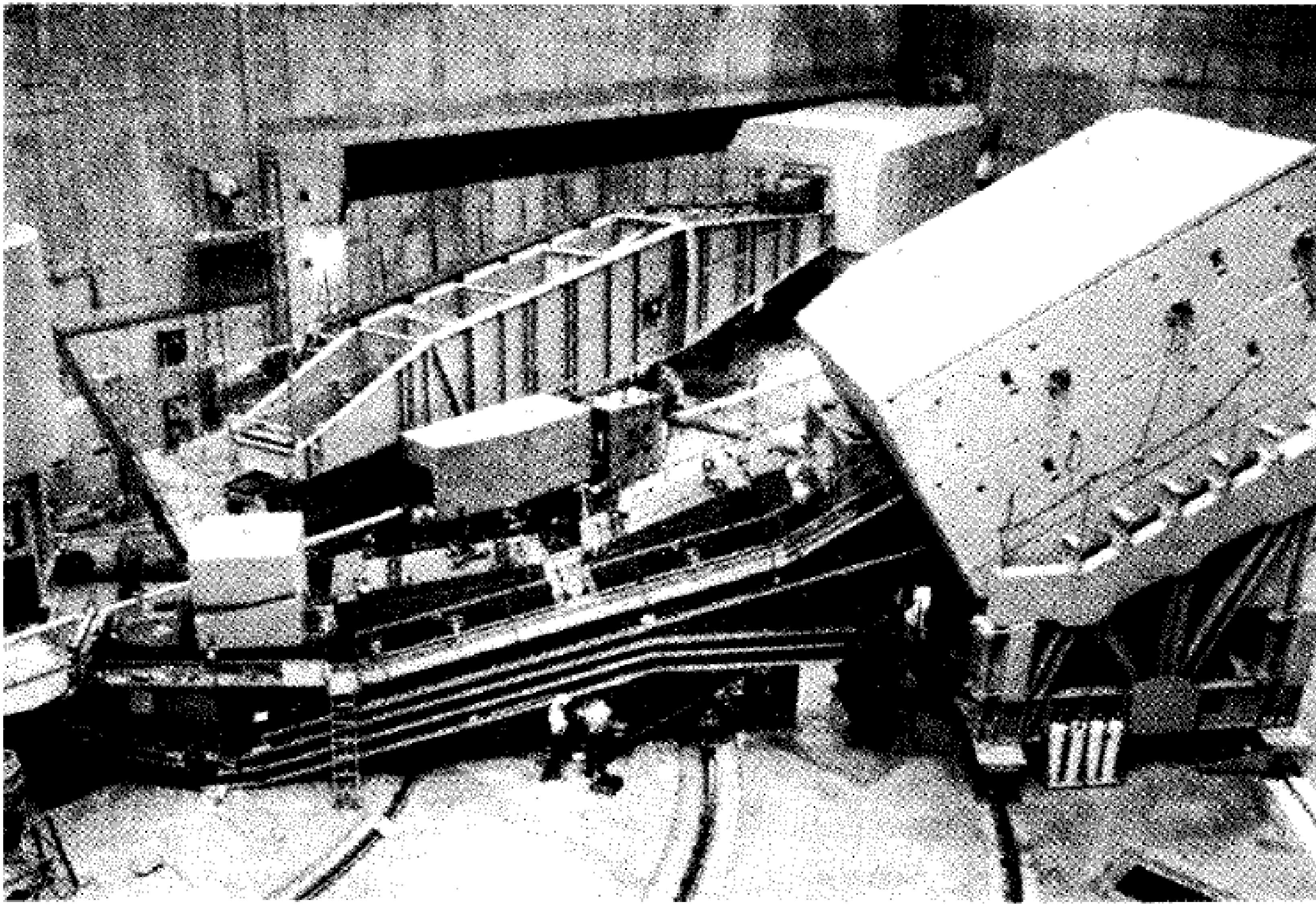


Fig. 20. Photograph of the 8 and 20 GeV spectrometers in End Station A.

# Part 2; Deep Inelastic Scattering

The reaction equation of deep inelastic scattering (DIS) is written

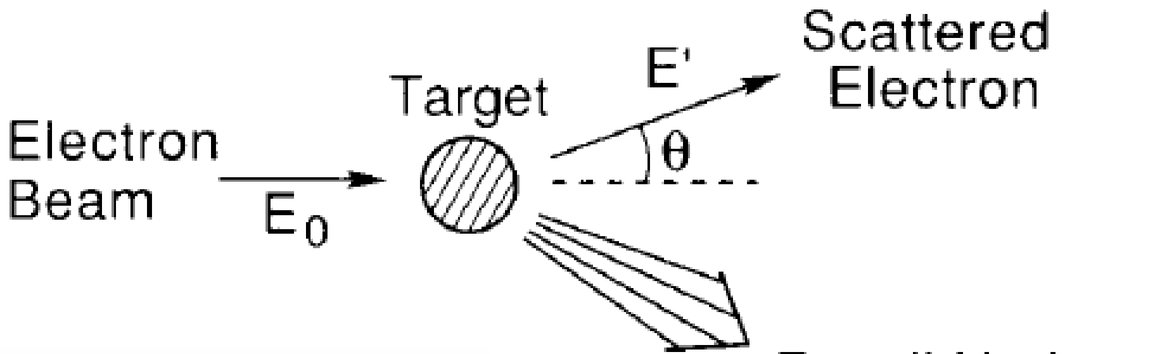
$$\boxed{e + p \rightarrow e + X}$$

where $X$ is a system of outgoing hadrons (mostly pions).
Observed is only the scattered electron.
The unobserved hadronic system is the ***missing mass***.

The energy of the incident electron beam is accurately known
The proton is the target particle.

In the SLAC experiments (and many later experiments at CERN)
the target is at rest in the laboratory.
This defines the LAB frame.

# DIS Kinematics:




$$Q^2 = 4E_0E' \sin^2\theta/2$$

$$E' = \frac{E_0 - \dfrac{(W^2 - M^2)}{2M}}{1 + \dfrac{2E_0}{M}\sin^2\theta/2}$$

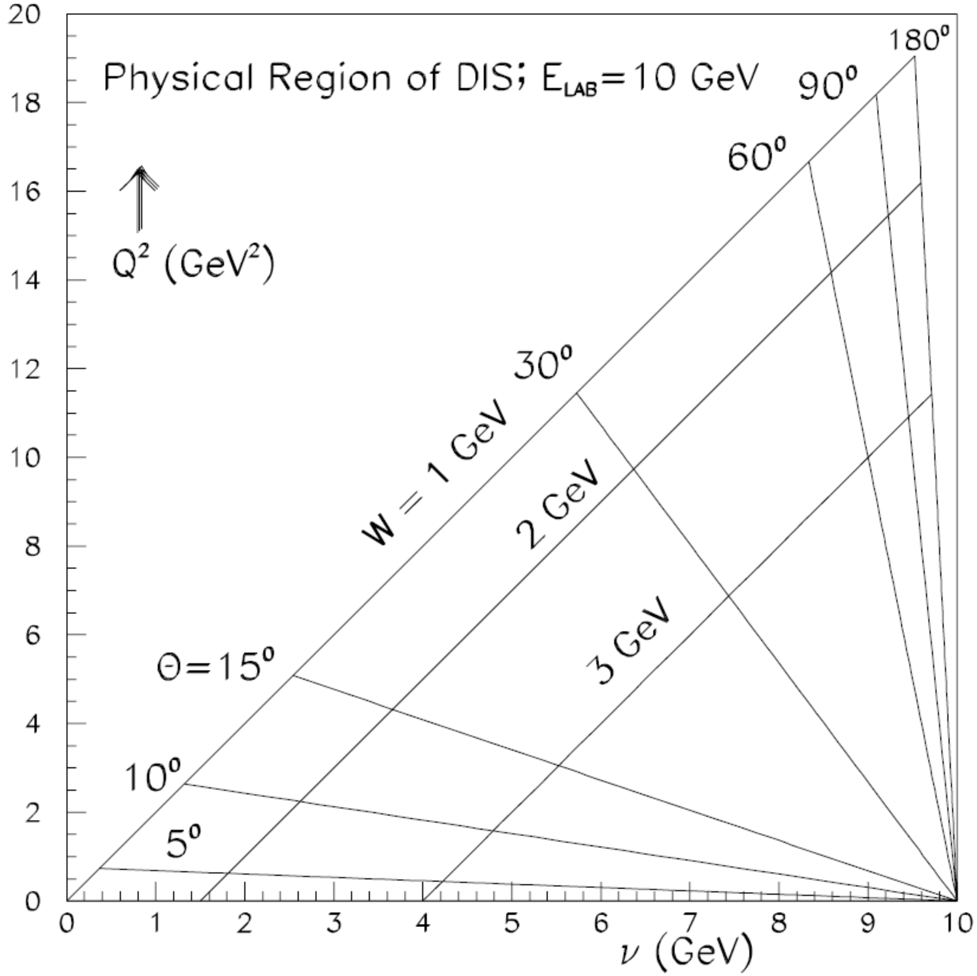
or, since $E'$ and $\theta$ are measured:

$$\boxed{W^2 = M^2 + 2M\left(E_0 - E'\right) - 4E_0E'\sin^2\frac{\theta}{2}}$$

Note: for elastic scattering $W=M$

## Physical Region of Deep Inelastic Scattering in the ($\nu$,$Q^2$) Plane

For the purpose of illustration a LAB energy of 10 GeV has been chosen, appropriate for the early work at SLAC when the parton structure of the nucleon was discovered. The proton mass was set = 1 GeV/$c^2$




$$\nu = E - E'$$

is the energy transfer from the electron to the proton.

At $\nu$=$E$ the entire energy is transferred to the proton;

at $\nu$=$0$ no energy is transferred.

# Feynman diagram of inelastic electron-proton scattering:

(from H.W. Kendall, “Deep Inelastic Scattering”, Nobel Lecture, December 8, 1990)

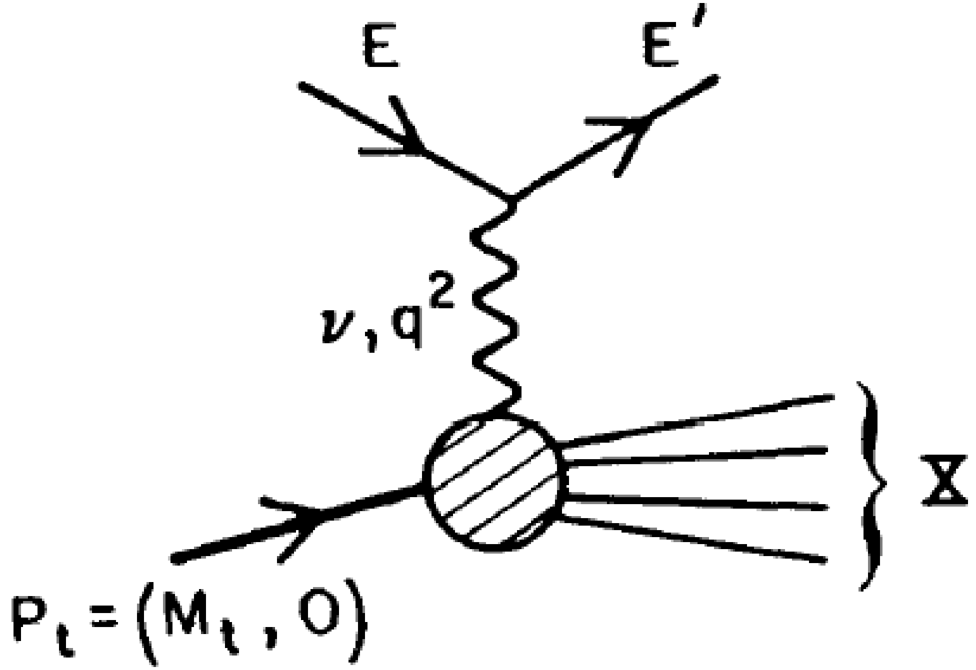



Fig. 5. Feynman diagram for inelastic electron scattering.

# Differential Cross Section of Inelastic ep Scattering

From relativistic quantum mechanics one gets the following formula:

$$\boxed{\frac{d\sigma}{d\Omega dE'} = \frac{\alpha^2}{4E_0^2 \sin^2 \theta/2} \cos^2 \theta/2 \, [W_2 + 2W_1 \tan^2 \theta/2]}$$

$W_{1,2} = W_{1,2}\left(Q^2, \nu\right)$ structure functions

$\nu = E_0 - E'$ is the energy transfer.

On general grounds the structure functions are functions of two kinematical invariants

They were expected to drop with increasing $Q^2$ as rapidly as the form factors ("*mushy proton*"!)

*The surprising experimental result was different!*

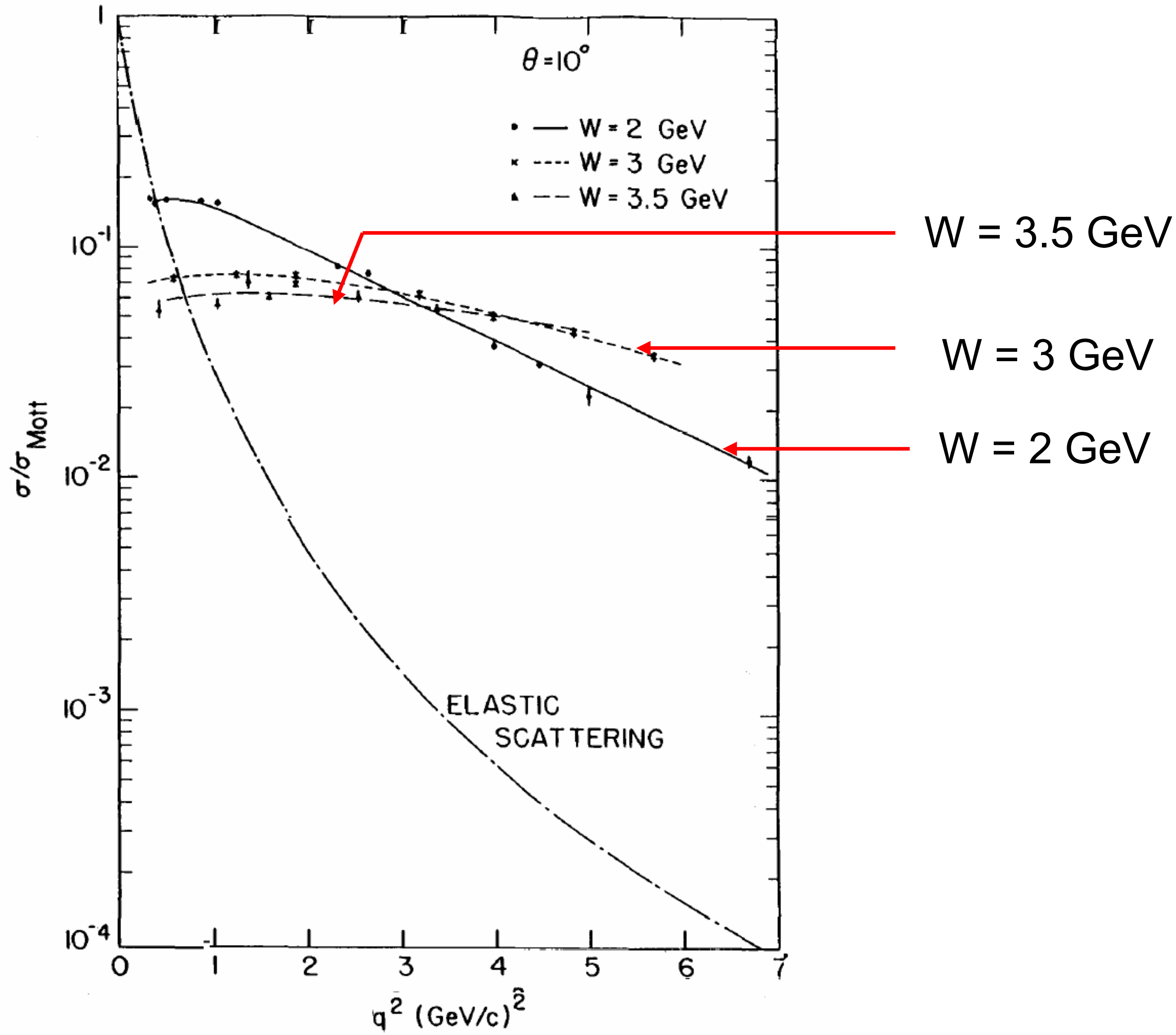
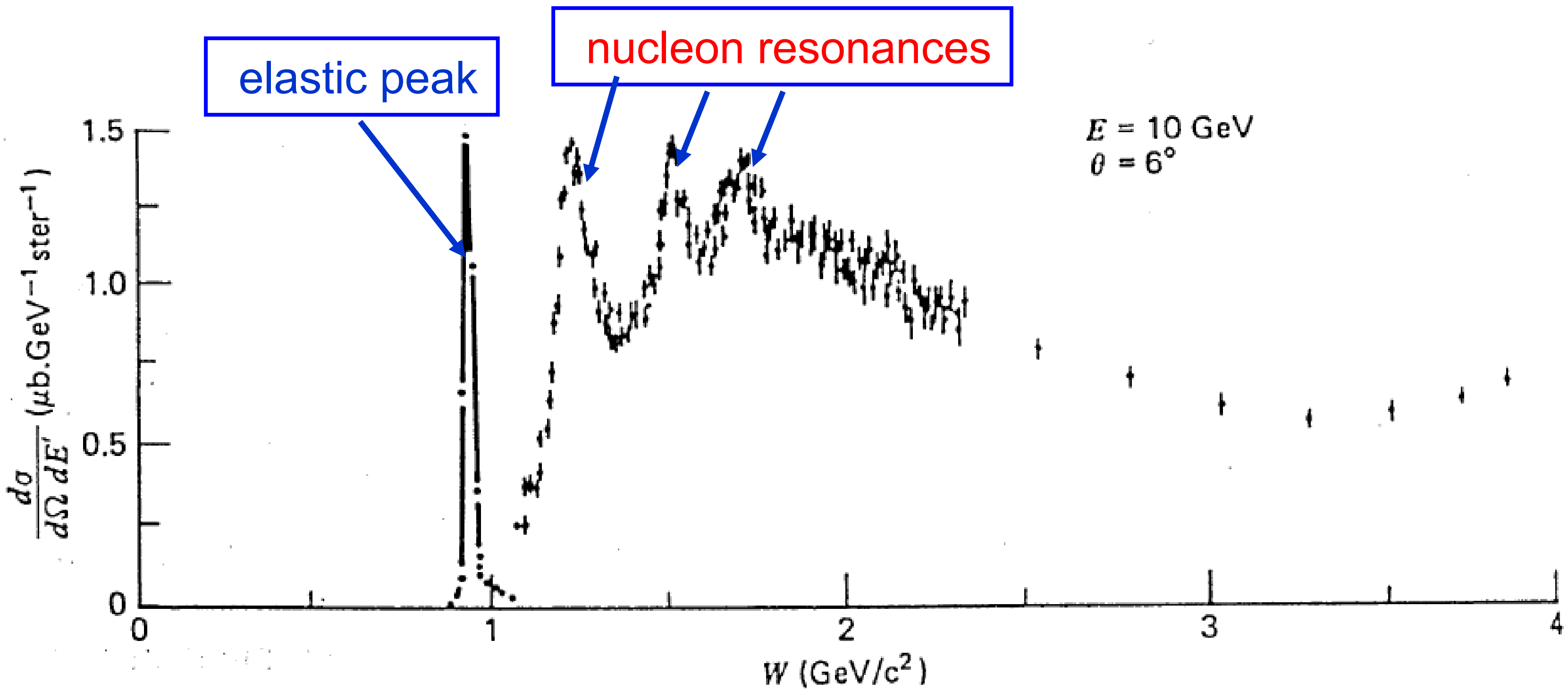



*Fig. 1:* $(d^2\sigma/d\Omega dE')/\sigma_{\text{Mott}}$, in $\text{GeV}^{-1}$, vs. $q^2$ for $W = 2$, 3 and 3.5 GeV. The lines drawn through the data are meant to guide the eye. Also shown is the cross section for elastic e-p scattering divided by $\sigma_{\text{Mott}}$, $(d\sigma/d\Omega)/\sigma_{\text{Mott}}$, calculated for $\theta = 10°$, using the dipole form factor. The relatively slow variation with $q^2$ of the inelastic cross section compared with the elastic cross section is clearly shown.

The Structure of Hadrons




up to about $W$ = 1.8 GeV there is structure corresponding to the production of resonances (excited nucleon states); there is no structure above 1.8 GeV: this is the region of DIS.

Shown here is the electromagnetic structure function $\nu W_2(Q_2,\nu)$ as a function of $Q^2$ for $\omega = 2, 3$, and $4$; the absence of a $Q^2$ dependence is called ***scaling*** ($\omega = 2M\nu/Q^2$).

Fig. 12.11. Scaling behaviour of electromagnetic structure function $\nu W_2$ at various $\omega$ values. There is virtually no variation with $Q^2$. (From Panofsky, 1968.)

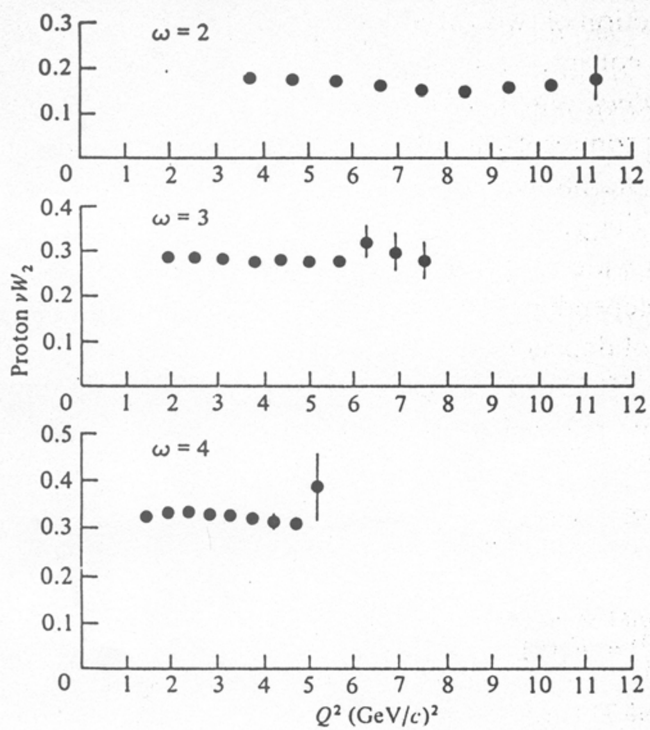
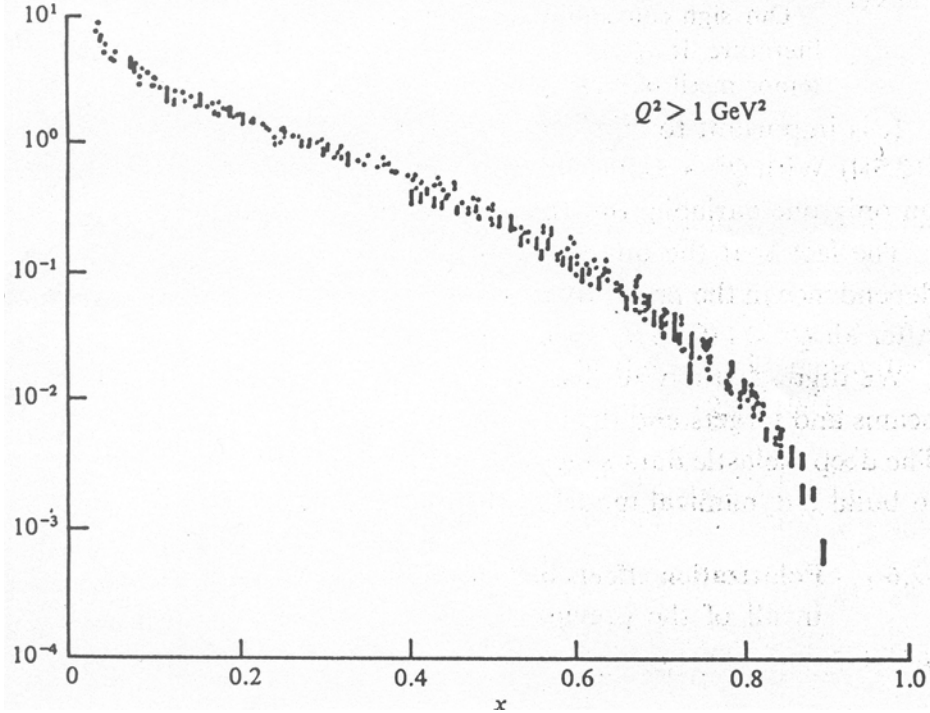


Fig. 12.12. Scaling behaviour of electromagnetic structure function $2m_N W_1$. Almost no $Q^2$ dependence is visible. (From Panofsky, 1968.)




Plotted along the abscissa is the scaling variable $x=1/\omega$ which is generally used today in preference to $\omega$.

## **Scaling** (Bjorken (1969)):

$$\boxed{\begin{array}{l}\text{for}\quad Q^2 \to \infty \ \text{ and } \ \nu \to \infty, \\ \text{such that } \omega = 2M\nu / Q^2 \text{ is fixed}\end{array}}$$

(**"Bjorken limit"**), the structure functions depend only on ***ω***:

$$\boxed{\begin{aligned} 2MW_1\left(Q^2,\nu\right) &\to F_1(\omega) \\ \nu W_2\left(Q^2,\nu\right) &\to F_2(\omega) \end{aligned}}$$

this behaviour is called ***scaling***; ***ω*** is the *scaling variable*

Scaling found a natural explanation in the *parton model* (Feynman).

Partons are constituents of the proton
(more generally of hadrons).

They are point-like fermions like the leptons.

But unlike the leptons they take part in strong interactions as well as electromagnetic and weak interactions.

(Leptons take part only in electromagnetic and weak interactions)

Today the Feynman partons are understood to be identical with the quarks postulated by Gell-Mann.

# Parton model picture of DIS:

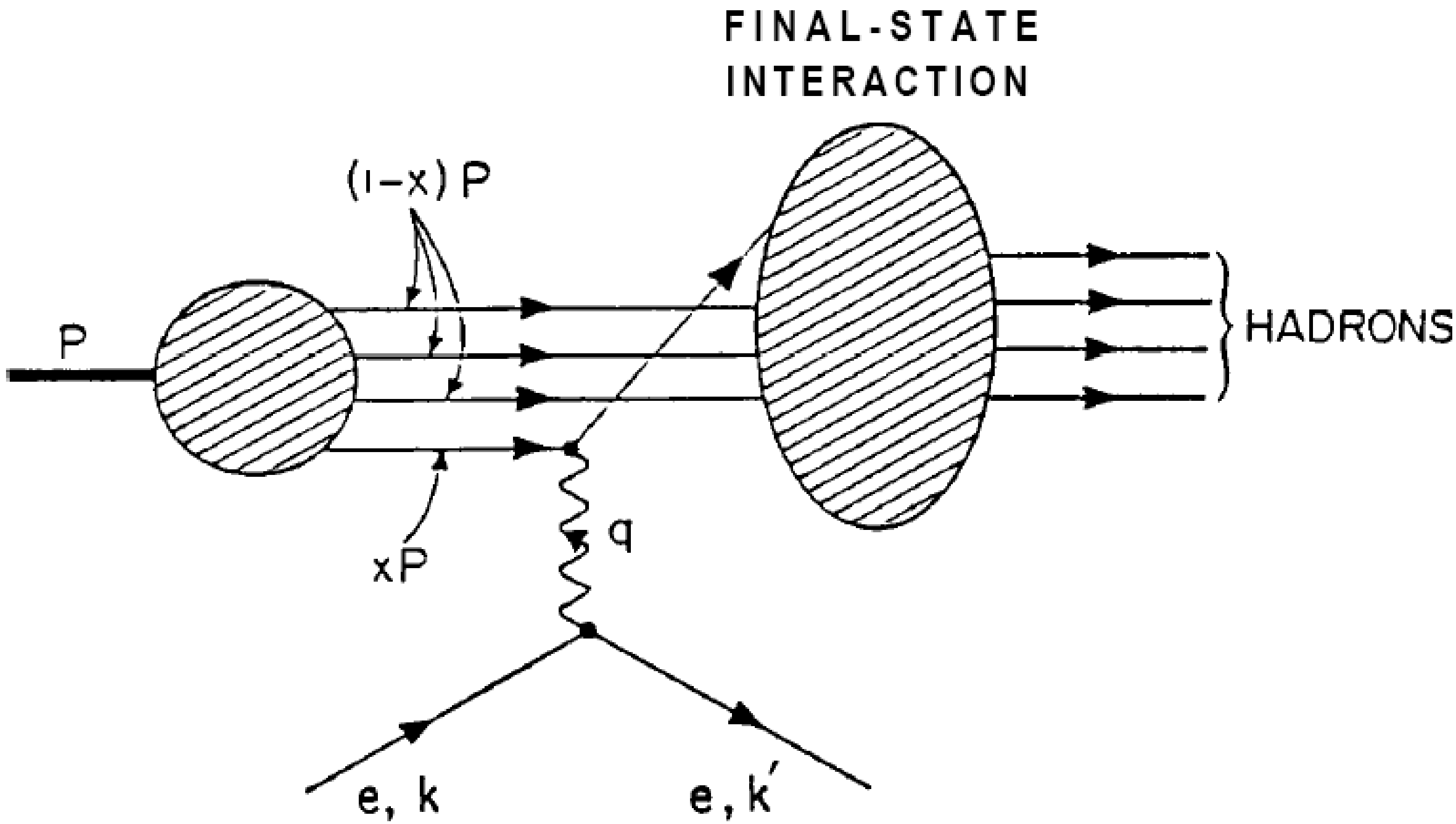



Fig. 4: A representation of inelastic electron nucleon scattering in the parton model. *k* and *k'* are the incident and final momenta of the electron. The other quantities are defined in the text.

The electron collides elastically with a **parton** that carries a fraction $x$ of the proton momentum.

At high momentum ("infinite" momentum) the partons are free. Therefore the collision of one parton with the electron does not affect the other partons. This leads to scaling in $\boldsymbol{x}$

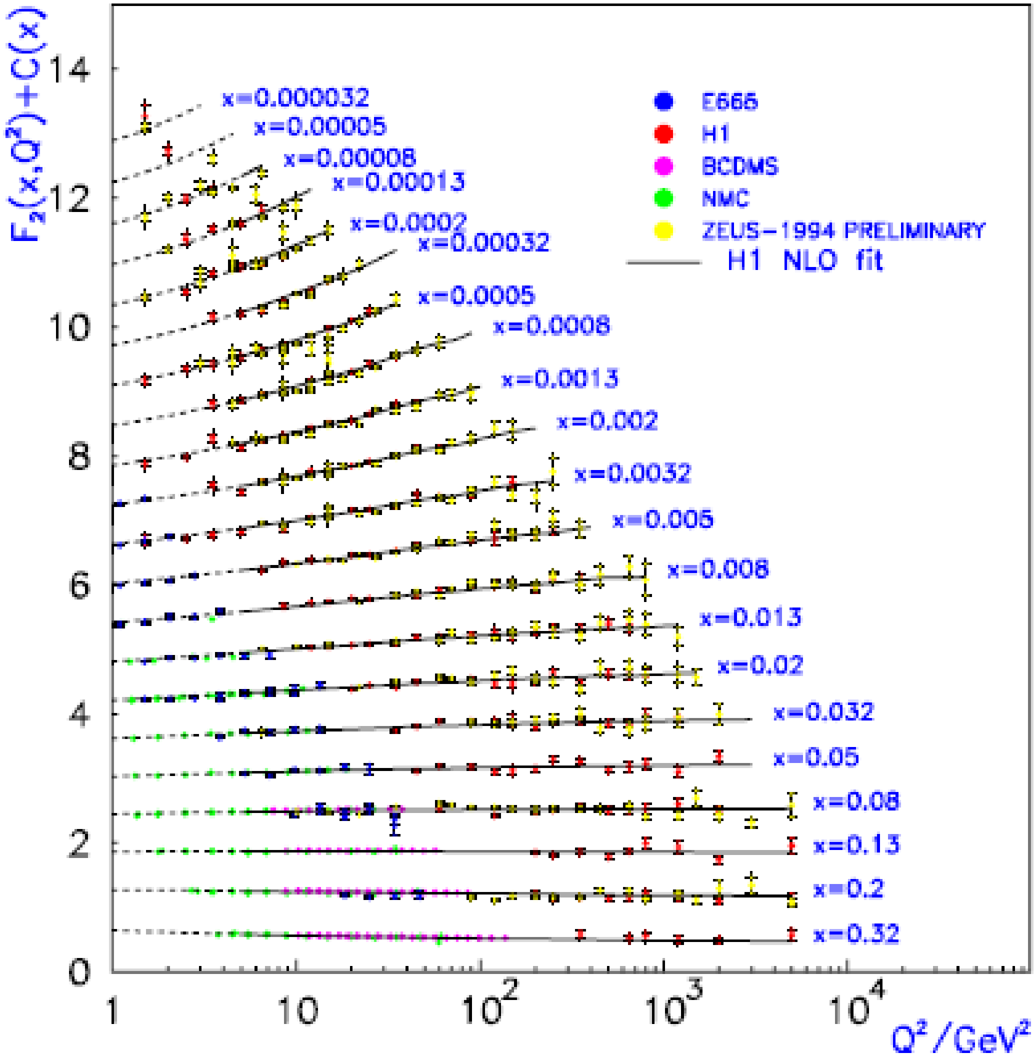
$$\boxed{x = 1/\omega = Q^2/2M\nu \in [0,1]}$$

DIS experiments have also been done with muons and with neutrinos.

Since 1992 DIS experiments are done also on the electron-proton collider HERA.

The next slides contains a summery of results of measurements of the proton structure function $F_2(x)$ from several experiments:

Results on the proton structure function $F_2$ from experiments at CERN, Fermilab and DESY



## Discussion of the results of measurements of $F_2(x,Q^2)$:

To separate the sets of data at different values of $x$ an offset function $C(x)$ has been added to the data. *This has no physical significance*.

The important physics result lies in the $Q^2$ dependence of the data: at $x$ values above about 0.05 the structure function $F_2$ is seen to be independent of $Q^2$, *i.e.* to show scaling behaviour.

At smaller values of $x$ there is a noticeable, and at very small values of $x$ a strong violation of scaling.

Since scaling requires the partons to behave like "free" particles, therefore scaling violation indicates the effect of binding on the partons. This is understood to be the result of the colour force transmitted by gluons.